Web Appendix of
"Measuring Ambiguity Attitudes for All (Natural) Events"

Aurélien Baillon, Zhenxing Huang, Asli Selim, & Peter P. Wakker

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Appendix WA Additional analysis of the source method

We replicate Theorem 3.5, and then give a detailed proof. AD abbreviates Abdellaoui et al. (2011) and Dimmock, Kouwenberg, & Wakker (2016).

THEOREM 3.5. Assume that a, b, p_1, p_2 have been chosen such that Eq. 3.4 best fits the data by quadratic distance and that weak monotonicity holds. Then our index a is identical to AD's index a'.

PROOF. We consider the following optimization problem:

$$\min_{\tau,\sigma,p_1,p_2}(m_1-\tau-\sigma p_1)^2+(m_2-\tau-\sigma p_2)^2+\big(m_3-\tau-\sigma(1-p_1-p_2)\big)^2$$
$$+\big(m_{23}-\tau-\sigma(1-p_1)\big)^2+\big(m_{13}-\tau-\sigma(1-p_2)\big)^2$$
$$+(m_{12}-\tau-\sigma(p_1+p_2))^2. \tag{WA.1}$$

We for now do not restrict the probabilities p_1, p_2 and allow them to be any real value, so that we can apply first order conditions to them. Weak monotonicity will imply that they are still probabilities; i.e., they are nonnegative and sum to less than 1.

The first order condition of Eq. WA.1 with respect to σ, divided by -2, gives Eq. A.3 from the main text, renumbered here:

$$p_1(m_1-\tau-\sigma p_1)+\ p_2(m_2-\tau-\sigma p_2)+\ p_3(m_3-\tau-\sigma p_3)+$$
$$p_{23}(m_{23}-\tau-\sigma p_{23})+p_{13}(m_{13}-\tau-\sigma p_{13})+$$

$$p_{12}(m_{12} - \tau - \sigma p_{12}) = 0. \tag{WA.2}$$

The case $\sigma = 0$ was handled in the main text, and therefore we assume Eq. A.4 of the main text, renumbered here:

$$\sigma \neq 0. \tag{WA.3}$$

To substitute the probabilities in Eq. WA.2, we consider the first order condition for p_1, divided by -2σ:

$$m_1 - \sigma p_1 - m_3 + \sigma(1 - p_1 - p_2) - m_{23} + \sigma(1 - p_1)$$
$$+ m_{12} - \sigma(p_1 + p_2) = 0 \,. \tag{WA.4}$$

That is,

$$m_1 - m_3 - m_{23} + m_{12} + 2\sigma - 2\sigma p_2 = 4\sigma p_1$$
$$4p_1 = \frac{m_1 - m_3 - m_{23} + m_{12} + 2\sigma - 2\sigma p_2}{\sigma} = \frac{m_1 - m_3 - m_{23} + m_{12}}{\sigma} + 2 - 2p_2 \,.$$

Similarly,

$$2p_2 = \frac{m_2 - m_3 - m_{13} + m_{12}}{2\sigma} + 1 - p_1 \,.$$

We substitute the latter formula into the preceding one:

$$4p_1 = \frac{2m_1 - 2m_3 - 2m_{23} + 2m_{12}}{2\sigma} + 2 - \frac{m_2 - m_3 - m_{13} + m_{12}}{2\sigma} - 1 + p_1 \,.$$
$$3p_1 = \frac{2m_1 - m_2 - m_3 - 2m_{23} + m_{13} + m_{12} + 2\sigma}{2\sigma}.$$
$$3p_1 = \frac{m_{13} + m_{12} + m_{23} - m_1 - m_2 - m_3 + 3m_1 - 3m_{23} + 2\sigma}{2\sigma}.$$

$$p_1 = \frac{3(\overline{m_c} - \overline{m_s}) + 3m_1 - 3m_{23} + 2\sigma}{6\sigma} \tag{WA.5}$$

(which is Eq. A.6 in the main text) and similarly Eq. A.7 of the main text is here:

$$p_2 = \frac{3(\overline{m_c} - \overline{m_s}) + 3m_2 - 3m_{13} + 2\sigma}{6\sigma} \,. \tag{WA.6}$$

Because $p_3 = 1 - p_1 - p_2$, we get Eq. A.8 from the main text:

$$p_3 = \frac{3(\overline{m_c} - \overline{m_s}) + 3m_3 - 3m_{12} + 2\sigma}{6\sigma} \,. \tag{WA.7}$$

Adding the three probabilities gives $\frac{6\sigma}{6\sigma} = 1$, as should be. Substituting Eqs. WA.5-WA.7 in Eq. WA.2 and multiplying by 6σ gives:

$$(3(\overline{m_c}-\overline{m_s})+3m_1-3m_{23}+2\sigma)\left(m_1-\tau-\frac{3(\overline{m_c}-\overline{m_s})+3m_1-3m_{23}+2\sigma}{6}\right)+$$
$$(3(\overline{m_c}-\overline{m_s})+3m_2-3m_{13}+2\sigma)\left(m_2-\tau-\frac{3(\overline{m_c}-\overline{m_s})+3m_2-3m_{13}+2\sigma}{6}\right)+$$
$$(3(\overline{m_c}-\overline{m_s})+3m_3-3m_{12}+2\sigma)\left(m_3-\tau-\frac{3(\overline{m_c}-\overline{m_s})+3m_3-3m_{12}+2\sigma}{6}\right)+$$
$$(4\sigma-3(\overline{m_c}-\overline{m_s})+3m_{23}-3m_1)\left(m_{23}-\tau-\frac{4\sigma-3(\overline{m_c}-\overline{m_s})-3m_1+3m_{23}}{6}\right)+$$
$$(4\sigma-3(\overline{m_c}-\overline{m_s})+3m_{13}-3m_2)\left(m_{13}-\tau-\frac{4\sigma-3(\overline{m_c}-\overline{m_s})-3m_2+3m_{13}}{6}\right)+$$
$$(4\sigma-3(\overline{m_c}-\overline{m_s})+3m_{12}-3m_3)\left(m_{12}-\tau-\frac{4\sigma-3(\overline{m_c}-\overline{m_s})-3m_3+3m_{12}}{6}\right)$$
$$=0.$$

Multiplying by 6:

$$(3(\overline{m_c}-\overline{m_s})+3m_1-3m_{23}+2\sigma)(3m_1-6\tau-3(\overline{m_c}-\overline{m_s})+3m_{23}-2\sigma))+$$
$$(3(\overline{m_c}-\overline{m_s})+3m_2-3m_{13}+2\sigma)(3m_2-6\tau-3(\overline{m_c}-\overline{m_s})+3m_{13}-2\sigma)+$$
$$(3(\overline{m_c}-\overline{m_s})+3m_3-3m_{12}+2\sigma)(3m_3-6\tau-3(\overline{m_c}-\overline{m_s})+3m_{12}-2\sigma)+$$
$$(4\sigma-3(\overline{m_c}-\overline{m_s})-3m_1+3m_{23})(3m_{23}-6\tau-4\sigma+3(\overline{m_c}-\overline{m_s})+3m_1)+$$
$$(4\sigma-3(\overline{m_c}-\overline{m_s})-3m_2+3m_{13})(3m_{13}-6\tau-4\sigma+3(\overline{m_c}-\overline{m_s})+3m_2)+$$
$$(4\sigma-3(\overline{m_c}-\overline{m_s})-3m_3+3m_{12})(3m_{12}-6\tau-4\sigma+3(\overline{m_c}-\overline{m_s})+3m_3)$$
$$=0.$$

We substitute $\tau=(\overline{m_c}+\overline{m_s}-\sigma)/2$ from Eq. A.2:

$$(3(\overline{m_c}-\overline{m_s})+3m_1-3m_{23}+2\sigma)(3m_1+3m_{23}-6\overline{m_c}+\sigma)+$$
$$(3(\overline{m_c}-\overline{m_s})+3m_2-3m_{13}+2\sigma)(3m_2+3m_{13}-6\overline{m_c}+\sigma)+$$
$$(3(\overline{m_c}-\overline{m_s})+3m_3-3m_{12}+2\sigma)(3m_3+3m_{12}-6\overline{m_c}+\sigma)+$$
$$(4\sigma-3(\overline{m_c}-\overline{m_s})-3m_1+3m_{23})(3m_{23}+3m_1-6\overline{m_s}-\sigma)+$$
$$(4\sigma-3(\overline{m_c}-\overline{m_s})-3m_2+3m_{13})(3m_{13}+3m_2-6\overline{m_s}-\sigma)+$$
$$(4\sigma-3(\overline{m_c}-\overline{m_s})-3m_3+3m_{12})(3m_{12}+3m_3-6\overline{m_s}-\sigma)$$
$$=0.$$

It implies:

$$3(\overline{m_c}-\overline{m_s})(9\overline{m_s}+9\overline{m_c}-18\overline{m_c}+3\sigma-9\overline{m_c}-9\overline{m_s}+18\overline{m_s}+3\sigma)$$
$$+9m_1^2+9m_1m_{23}-18m_1\overline{m_c}+3m_1\sigma-9m_1m_{23}-9m_1^2+18m_1\overline{m_s}+3m_1\sigma$$

$+9m_2^2 + 9m_2m_{13} - 18m_2\overline{m_c} + 3m_2\sigma - 9m_2m_{13} - 9m_2^2 + 18m_2\overline{m_s} + 3m_2\sigma$

$+9m_3^2 + 9m_3m_{12} - 18m_3\overline{m_c} + 3m_3\sigma - 9m_3m_{12} - 9m_3^2 + 18m_3\overline{m_s} + 3m_3\sigma$

$-9m_1m_{23} - 9m_{23}^2 + 18m_{23}\overline{m_c} - 3m_{23}\sigma + 9m_{23}^2 + 9m_1m_{23} - 18m_{23}\overline{m_s} - 3m_{23}\sigma$

$-9m_2m_{13} - 9m_{13}^2 + 18m_{13}\overline{m_c} - 3m_{13}\sigma + 9m_{13}^2 + 9m_2m_{13} - 18m_{13}\overline{m_s} - 3m_{13}\sigma$

$-9m_3m_{12} - 9m_{12}^2 + 18m_{12}\overline{m_c} - 3m_{12}\sigma + 9m_{12}^2 + 9m_3m_{12} - 18m_{12}\overline{m_s} - 3m_{12}\sigma$

$+6m_1\sigma + 6m_{23}\sigma - 12\overline{m_c}\sigma + 2\sigma^2$

$+6m_2\sigma + 6m_{13}\sigma - 12\overline{m_c}\sigma + 2\sigma^2$

$+6m_3\sigma + 6m_{12}\sigma - 12\overline{m_c}\sigma + 2\sigma^2$

$+12m_{23}\sigma + 12m_1\sigma - 24\overline{m_s}\sigma - 4\sigma^2$

$+12m_{13}\sigma + 12m_2\sigma - 24\overline{m_s}\sigma - 4\sigma^2$

$+12m_{12}\sigma + 12m_3\sigma - 24\overline{m_s}\sigma - 4\sigma^2$

$= 0.$

That is,

$3(\overline{m_c} - \overline{m_s})(-18\overline{m_c} + 18\overline{m_s} + 6\sigma)$

$-18m_1(\overline{m_c} - \overline{m_s}) + 6m_1\sigma$

$-18m_2(\overline{m_c} - \overline{m_s}) + 6m_2\sigma$

$-18m_3(\overline{m_c} - \overline{m_s}) + 6m_3\sigma$

$+18m_{23}(\overline{m_c} - \overline{m_s}) - 6m_{23}\sigma$

$+18m_{13}(\overline{m_c} - \overline{m_s}) - 6m_{13}\sigma$

$+18m_{12}(\overline{m_c} - \overline{m_s}) - 6m_{12}\sigma$

$+6m_1\sigma + 6m_{23}\sigma - 12\overline{m_c}\sigma + 2\sigma^2$

$+6m_2\sigma + 6m_{13}\sigma - 12\overline{m_c}\sigma + 2\sigma^2$

$+6m_3\sigma + 6m_{12}\sigma - 12\overline{m_c}\sigma + 2\sigma^2$

$+12m_{23}\sigma + 12m_1\sigma - 24\overline{m_s}\sigma - 4\sigma^2$

$+12m_{13}\sigma + 12m_2\sigma - 24\overline{m_s}\sigma - 4\sigma^2$

$+12m_{12}\sigma + 12m_3\sigma - 24\overline{m_s}\sigma - 4\sigma^2$

$= 0,$

implying

$3(\overline{m_c} - \overline{m_s})(6\sigma) + 6m_1\sigma + 6m_2\sigma + 6m_3\sigma - 6m_{23}\sigma - 6m_{13}\sigma - 6m_{12}\sigma$

$+6m_1\sigma + 6m_{23}\sigma - 12\overline{m_c}\sigma + 2\sigma^2$

$+6m_2\sigma + 6m_{13}\sigma - 12\overline{m_c}\sigma + 2\sigma^2$

$+6m_3\sigma + 6m_{12}\sigma - 12\overline{m_c}\sigma + 2\sigma^2$

$+12m_{23}\sigma + 12m_1\sigma - 24\overline{m_s}\sigma - 4\sigma^2$

$+12m_{13}\sigma + 12m_2\sigma - 24\overline{m_s}\sigma - 4\sigma^2$

$+12m_{12}\sigma + 12m_3\sigma - 24\overline{m_s}\sigma - 4\sigma^2$

$= 0,$

or, combining terms σ^2 and dividing by 6,

$3(\overline{m_c} - \overline{m_s})\sigma + m_1\sigma + m_2\sigma + m_3\sigma - m_{23}\sigma - m_{13}\sigma - m_{12}\sigma$

$+m_1\sigma + m_{23}\sigma - 2\overline{m_c}\sigma$

$+m_2\sigma + m_{13}\sigma - 2\overline{m_c}\sigma$

$+m_3\sigma + m_{12}\sigma - 2\overline{m_c}\sigma$

$+2m_{23}\sigma + 2m_1\sigma - 4\overline{m_s}\sigma$

$+2m_{13}\sigma + 2m_2\sigma - 4\overline{m_s}\sigma$

$+2m_{12}\sigma + 2m_3\sigma - 4\overline{m_s}\sigma - \sigma^2$

$= 0,$

which yields

$\sigma(3\overline{m_c} - 3\overline{m_s} - \sigma) = 0.$

Eq. WA.3 precludes $\sigma = 0$, and therefore the solution is

$$\sigma = 3\overline{m_c} - 3\overline{m_s},$$

implying $a' = 1 - \sigma = 1 - (3\overline{m_c} - 3\overline{m_s}) = a$.

In the appendix in the main text it is shown that $\sigma > 0$ and that the p_j's are probabilities, which concludes the proof. □

The following lemma may be useful. It considers optimization w.r.t. σ, τ, without assuming anything about the probabilities (other than that they add to 1).

LEMMA WA.1. To minimize the distance in Eq. A.1, the sign of σ is equal to the sign of the correlation between the probabilities p and their m values.

PROOF. We reproduce the distance to be minimized from Eq. A.1:

$$(m_1 - \tau - \sigma p_1)^2 + (m_2 - \tau - \sigma p_2)^2 + (m_3 - \tau - \sigma p_3)^2$$
$$+(m_{23} - \tau - \sigma p_{23})^2 + (m_{13} - \tau - \sigma p_{13})^2 + (m_{12} - \tau - \sigma p_{12})^2 \ .$$

The average value of the m's, i.e. $(\overline{m_c} + \overline{m_s})/2$, is denoted $\overline{m}$. Eq. A.2 implies $\tau = \overline{m} - \sigma/2$. Substituting this in Eq. WA.2 implies

$$\sigma = \frac{p_1 m_1 + \ p_2 m_2 + p_3 m_3 + p_{23} m_{23} + \ p_{13} m_{13} + p_{12} m_{12} - 3\overline{m}}{({p_1}^2 + {p_2}^2 + {p_3}^2 + {p_{23}}^2 + {p_{13}}^2 + {p_{12}}^2 - 3/2)} .$$

The denominator is always positive, taking its minimum $(\frac{1}{6})$ at $p_1 = p_2 = 1/3$. The numerator is the covariance between the p's and m's. □

Appendix WB Results excluding violations of weak monotonicity

Weak monotonicity was violated was violated by 8 subjects in part 1 only, by 4 subjects in part 2 only, and by 2 subjects in both parts. Hence, excluding the indexes when weak monotonicity is violated results in dropping 16 observations. Tables WB.1 and WB.2 below are equivalent to Tables 3 and 4 of the paper when these 16 observations are dropped. Unlike in Table 3, part 2 * control treatment (capturing learning effects in the control treatment) is not significant anymore.

TABLE WB.1: ambiguity aversion indexes *b*

	Model 1	Model 2
intercept	-0.07*	0.03
	(0.04)	(0.06)
part 1 * TP treatment	**-0.03**	**-0.04**
	(0.05)	**(0.05)**
part 2 * control treatment	-0.04	-0.04
	(0.02)	(0.02)
part 2 * TP treatment	-0.01	-0.02
	(0.05)	(0.05)
male		-0.09*
		(0.04)
Dutch		-0.08
		(0.05)
age – 20		0.01
		(0.01)
Chi2	3.89	14.40*
N	182	182

$^{+}$ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Point estimates are followed by standard errors between brackets. The impact of TP is in bold. The variable age has been recoded as age − 20 so that the intercept corresponds to the *b* index of a 20 year-old subject (median age)

TABLE WB.2: a-insensitivity indexes a

	Model 1	Model 2
intercept	0.10	0.20*
	(0.06)	(0.10)
part 1 * TP treatment	**0.19***	**0.18***
	(0.08)	**(0.09)**
part 2 * control treatment	0.06	0.06
	(0.05)	(0.05)
part 2 * TP treatment	0.01	0.00
	(0.08)	(0.09)
male		-0.08
		(0.08)
Dutch		-0.08
		(0.09)
age – 20		0.01
		(0.02)
Chi2	17.72***	20.24**
N	182	182

$^{+}$ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Point estimates are followed by standard errors between brackets. The impact of TP is in bold. The variable age has been recoded as age − 20 so that the intercept corresponds to the a index of a 20 year-old subject (median age)

Appendix WC Impact of TP on index *b*

We tested whether TP would make index *b* more or less extreme by running the same regressions as described in the main text on the absolute value of *b*. TP did not have any impact on the magnitude of *b*.

TABLE WC.1: absolute value of the ambiguity aversion indexes *b*

	Model 1	Model 2
intercept	0.18***	0.15***
	(0.02)	(0.04)
part 1 * TP treatment	**0.01**	**0.01**
	(0.03)	**(0.03)**
part 2 * control treatment	0.02	0.02
	(0.02)	(0.02)
part 2 * TP treatment	0.02	0.02
	(0.03)	(0.03)
male		0.02
		(0.03)
Dutch		0.02
		(0.03)
age – 20		0.00
		(0.01)
Chi2	1.19	1.94
N	198	198

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Point estimates are followed by standard errors between brackets. The impact of TP is in bold. The variable age has been recoded as age − 20 so that the intercept corresponds to the *b* index of a 20 year-old subject (median age)