MEASURING AMBIGUITY ATTITUDES FOR ALL (NATURAL) EVENTS

AURÉLIEN BAILLON[a], ZHENXING HUANG[b], ASLI SELIM[a], & PETER P. WAKKER[a]

a: Erasmus School of Economics, Erasmus University Rotterdam, Rotterdam, the Netherlands, Baillon@ese.eur.nl, Wakker@ese.eur.nl; asliselim@gmail.com

b: School of Economics, Shanghai University of Finance and Economics, and Key Laboratory of Mathematical Economics (SUFE), Ministry of Education, Shanghai, P.R. China, ybhzxadam@gmail.com

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Measurements of ambiguity attitudes have so far focused on artificial events, where (subjective) beliefs can be derived from symmetry of events and can be then controlled for. For natural events as relevant in applications, such a symmetry and corresponding control are usually absent, precluding traditional measurement methods. This paper introduces two indexes of ambiguity attitudes, one for aversion and the other for insensitivity/perception, for which we can control for likelihood beliefs even if these are unknown. Hence, we can now measure ambiguity attitudes for natural events. Our indexes are valid under many ambiguity theories, do not require expected utility for risk, and are easy to elicit in practice. We use our indexes to investigate time pressure under ambiguity. People do not become more ambiguity averse under time pressure but become more insensitive (perceive more ambiguity). These findings are plausible and, hence, support the validity of our indexes.

JEL-CLASSIFICATION: D81, C91

KEYWORDS: ambiguity aversion; Ellsberg paradox; sources of uncertainty; time pressure

1. INTRODUCTION

Ambiguity (unknown probabilities) is central in many practical decisions (Keynes 1921; Knight 1921). Ellsberg's paradox (1961) showed that fundamentally new models are needed to handle ambiguity. Gilboa (1987), Gilboa and Schmeidler (1989), and Schmeidler (1989) introduced such new models, with many to follow.[1] Ambiguity theories are now widely applied (Easley and O'Hara 2009; Guidolin and Rinaldi 2013; Shaw 2016). However, measurements of ambiguity have lagged behind, usually employing artificial laboratory events as in Ellsberg's paradox rather than the natural events that occur in practice.

To properly measure ambiguity aversion, we need to control for likelihood beliefs in the events of interest to calibrate the benchmark of ambiguity neutrality. But this control is difficult to implement for natural events. To illustrate this point, consider someone preferring to receive \$100 under the ambiguous event A of an increase in copper price of at least 0.01% next week rather than receiving \$100 under the risky event K of heads in a coin toss ($p = 0.5$) next week. This preference does not necessarily reflect ambiguity seeking. The person may be ambiguity neutral but assign a higher likelihood belief to A than to K. Without proper control of likelihoods, we cannot know people's ambiguity attitudes. However, it is still unclear how we can control for likelihoods of naturally occurring events using revealed preferences in a tractable manner.

Control for likelihoods can easily be obtained for artificial events generated in the lab. Such events may concern Ellsberg urns with color compositions kept secret from the subjects, or situations in which subjects are only informed about experimenter-specified probability intervals. Then likelihood beliefs can be derived from symmetry of colors or from symmetry about the midpoints of intervals. This explains why measurements of ambiguity have focused on such artificial events.

Several authors have warned against this focus on artificial ambiguities, arguing for the importance of natural events (Camerer and Weber 1992 p. 361; Ellsberg 2011

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[1] Theoretical surveys include Etner, Jeleva, and Tallon (2012), Gilboa and Marinacci (2016), Machina and Siniscalchi (2014), and Marinacci (2015).

p. 223; Heath and Tversky 1991 p. 6; Trautmann and van de Kuilen 2015 p. 94). The difficulty to identify (revealed preference based) likelihood beliefs of such events has been a problematic obstacle. This paper introduces a simple method to measure ambiguity attitudes for natural events. The solution to the problem is surprisingly easy: we do not control for likelihoods by directly measuring them but by making them drop from the equations irrespective of what they are. Our method is tractable and easy to implement, as we demonstrate in an experiment, and can easily be used as an add-on in large-scale surveys and field studies. Using natural events increases external validity (Camerer and Weber 1992 p. 361).

Empirical studies, discussed later, have shown that ambiguity is a rich phenomenon. Hence, two indexes are needed to capture ambiguity descriptively. The first measures the well-known aversion to ambiguity and is often taken to be normative. The second captures the degree of ambiguity, i.e., the perceived level of ambiguity. Dimmock et al. (2015) therefore called their version of this index perceived level of ambiguity. The higher this level is, the less the decision maker discriminates between different degrees of likelihood, and the more these degrees are treated alike, as one blur. The second index thus also captures insensitivity toward likelihood changes, which is why we use the term a(mbiguity generated)-insensitivity. Empirical studies have found that (uncorrected) ambiguity aversion is likelihood dependent, even with prevailing ambiguity seeking, rather than aversion, for unlikely gain-events (Trautmann and van de Kuilen 2015). That is, ambiguity aversion even predicts in the wrong direction for such events. This illustrates the desirability to use the second index to correct for this likelihood dependence.

To summarize, relative to their predecessors, our indexes: (a) correct for subjective likelihoods also if unknown; hence, which is our main novelty: (b) can be used for all events, both artificial and natural; (c) correct for likelihood dependence of ambiguity aversion. Further, as discussed later, our indexes (d) are directly observable; (e) are valid under many ambiguity theories, unifying preceding indexes; and they (f) retain validity if expected utility for risk is violated. Our paper also shows that the ambiguity aversion index can better be related to matching probabilities than to nonadditive weighting functions as was done before (Dow and Werlang 1992); see §5.

Our indexes are defined for three-fold partitions. The follow-up paper Baillon, Li, and Wakker (2018), a theoretical counterpart to this paper, provides a theoretical foundation of our indexes, and generalizations to general partitions. It shows that our indexes are valid for many ambiguity theories, being all that evaluate prospects $\gamma_E 0$ (yielding one nonzero outcome, γ, under event E, and 0 otherwise) by a product $W(E)U(\gamma)$.[2] This implies that we assume only one utility function U, for risk and all sources of uncertainty. Here we deviate from utility-based models of ambiguity, such as the popular smooth model (Klibanoff, Marinacci, and Mukerji 2005).[3] Baillon, Li, and Wakker (2018) show that our indexes are nevertheless also useful under the smooth model.[4] Our indexes thus unify and generalize many existing indexes (point (e) above). Baillon, Li, and Wakker (2018) also show that our two indexes capture orthogonal, i.e., completely distinct, components of the data. This mathematical separation supports the psychological interpretation of the indexes as distinct components and the empirical desirability to consider both. This paper will support these points empirically.

Because a-insensitivity is less known than ambiguity aversion, and different interpretations are possible for our insensitivity index, we test how our two indexes react to cognitive manipulations. For this purpose, we use time pressure (TP). TP has received special attention in the psychological literature because it provides a good context for manipulating cognitive limitations, in addition to its practical relevance (Ariely and Zakay 2001; Essl and Jaussi 2017). De Paola and Gioia (2015) and

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[2] As yet, indexes proposed in the literature concerned only one ambiguity theory. In the same way as no risk aversion index can be valid for all risk theories, our ambiguity indexes cannot be valid for all ambiguity theories. Yet, they are for many. Such theories include biseparable utility (Ghirardato and Marinaci 2002), which in turn includes Choquet expected utility (Schmeidler 1989), prospect theory for gains (Tversky and Kahneman 1992), and the α-maxmin model (Ghirardato, Maccheroni, and Marinacci 2004; Jaffray 1994). Some nonbiseparable theories that are included: separate-outcome weighting theories $\big(x_E y \to W(E)U(x) + W(E^c)U(y)$; Einhorn and Hogarth 1985$\big)$, Chateauneuf and Faro's (2009) confidence representation if the worst outcome is 0, and Lehrer and Teper's (2015) event-separable representation.

[3] Other utility-based theories not included are: Chew et al. (2008), Dobbs (1991), Nau (2006), and Neilson (2010). Further theories not included are: Chateauneuf and Faro (2009) in general, Gul and Pesendorfer (2014), Izhakian (2017), Olszewski (2007), and Siniscalchi (2009).

[4] That is, our indexes provide local ambiguity premiums in probability units that are analogous to Pratt's (1964 Eq. 5) local risk premium, which was in money units. This result also holds for a subclass of Maccheroni, Marinacci, and Rustichini's (2006) variational model.

Spiliopoulos and Ortmann (2017) argued for the usefulness of TP and, relatedly, response time, as a tool in experimental economics and for its relevance in economic applications. Despite the many studies of TP under risk (known probabilities; references are in §4.4), no studies have examined TP under ambiguity yet. Providing the first such study is an additional contribution of our paper. Our findings corroborate the interpretation of the indexes, supporting the validity of our method and illustrating the usefulness of our second index.

The outline of this paper is as follows. Section 2 gives formal definitions of our ambiguity indexes and informal arguments for their plausibility. We present the indexes without assuming any decision theory so that empirically oriented readers can readily use them with no need to study such a theory. This also shows that the indexes have intuitive appeal without requiring a commitment to one of the many ambiguity theories popular today. Sections 3-4 demonstrate the validity of our indexes empirically, and Sections 5-6 discuss and conclude. Experimental details are in the Appendix, with further details in an Online Appendix.

2. MEASURING AMBIGUITY ATTITUDES WITHOUT MEASURING SUBJECTIVE LIKELIHOODS: DEFINITIONS OF OUR INDEXES

We focus on gain outcomes throughout this paper. Formally speaking, ambiguity does not concern just a single event E, but a partition, such as $\{E, E^c\}$, or, more generally, a source of uncertainty. We assume a minimal degree of richness of the sources of uncertainty considered: there should be three mutually exclusive and exhaustive nonnull events E_1, E_2, and E_3. In many situations where we start from a partition with two events, we can extend it by properly partitioning one of those two events. For example, in the two-color Ellsberg urn we can involve other features of the ball to be drawn, such as shades of colors or numbers on the balls. The events in our experiment refer to the AEX stock index. For instance, in Part 1 of the experiment, $E_1 = (-100, -0.2), E_2 = [-0.2, 0.2]$, and $E_3 = (0.2, \infty)$, where intervals describe percentage increases of the AEX index during the experiment. They thus concern natural events with uncertainty that really occurred and that was of practical relevance to financial traders. E_{ij} denotes the union $E_i \cup E_j$, where $i \neq j$ is implicit. We call every E_i a *single event* and every E_{ij} a *composite event*.

Dimmock et al. (2016a, Theorem 3.1) showed that matching probabilities are convenient for measuring ambiguity attitudes. Early applications include Kahn and Sarin (1988) and Viscusi and Magat (1992). Matching probabilities entirely capture ambiguity attitudes, free from any complications regarding risk attitudes, because those drop from the equations and do not need to be measured. We therefore use matching probabilities. A drawback of matching probabilities is that their assessment is cognitively more difficult than, for instance, of certainty equivalents (Bleichrodt, Pinto, and Wakker 2001 p. 1505; Callen et al. 2013 p. 136; Halter and Beringer 1960 p. 124). However, an advantage is that they can be measured for all kinds of outcomes, also if nonquantitative. For any fixed prize (€20 in our experiment), we define the *matching probability* m of event E through the following indifference:

Receiving €20 under event E is equivalent to receiving €20 with probability m. (1)

In both prospects it is understood that the complementary payoff is nil. Under ambiguity neutrality, the matching probability of an event, say $m(E_1)$, and its complement, $m(E_{23})$, will add to 1, but under ambiguity aversion, the sum will fall below 1. The difference with 1 can then be taken as the degree of aversion. We take the average of this difference over the three events. We write $m_i = m(E_i)$, $m_{ij} = m(E_{ij})$, $\overline{m_s} = (m_1 + m_2 + m_3)/3$ for the average single-event matching probability, $\overline{m_c} = (m_{12} + m_{13} + m_{23})/3$ for the average composite-event matching probability, and define:

DEFINITION 2.1. The *ambiguity aversion index* is

$$b = 1 - \overline{m_c} - \overline{m_s}. \tag{2}$$

Note that no statistical claims or randomness assumptions are made at this stage in this definition. We use a deterministic calculation here, recoding direct observations. Under ambiguity neutrality[5], $m_i = P(E_i)$ and $m_{ij} = P(E_i) + P(E_j)$ for some additive subjective probability measure P. Then $\overline{m_s} = 1/3$ and $\overline{m_c} = 2/3$, implying $b = 0$. We have thus calibrated ambiguity neutrality, providing control for subjective likelihoods even though we do not know them. This happens because the subjective likelihoods drop from the equations irrespective of what they are. This observation is key to our method. Maximal ambiguity aversion occurs for $b = 1$. The matching probabilities of all events are then 0. Ambiguity aversion is minimal for $b = -1$, when matching probabilities for all events are 1.

The ambiguity aversion index can also be defined if we only consider a two-event partition. We can focus on only one event E_i and its complement E_i^c, and substitute $m(E_i)$ for $\overline{m_s}$ and $m(E_i^c)$ for $\overline{m_c}$ in Eq. 2, maintaining the control for likelihood. This would reduce the measurement effort—at the cost of reliability. However, for the insensitivity index defined next we need three events.

Theoretically, the second index captures the extent to which matching probabilities and event weights regress towards fifty-fifty, with low likelihoods overvalued and high likelihoods undervalued. This leads to reduced differences $\overline{m_c} - \overline{m_s}$. In the most extreme case of complete ambiguity and, correspondingly, complete insensitivity (Cohen and Jaffray 1980), no distinction is made between levels of likelihood (e.g., all events are taken as fifty-fifty), resulting in $\overline{m_c}$–$\overline{m_s}$ = 0. These observations suggest that the second index can be interpreted as a cognitive component (Budescu et al. 2014 p. 3; Dimmock et al. 2015; Dimmock et al 2016a; Einhorn and Hogarth 1985; Gayer 2010), an interpretation well supported by our results. For this index, the following rescaling of $\overline{m_c}$–$\overline{m_s}$ is convenient.

DEFINITION 2.2. The *ambiguity-generated insensitivity (a-insensitivity) index*[6] is

$$a = 3 \times (\tfrac{1}{3} - (\overline{m_c} - \overline{m_s}))\ . \tag{3}$$

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[5] When objective probabilities are assumed present in the domain considered, as is our case, then ambiguity neutrality is equivalent to probabilistic sophistication (Dean and Ortoleva 2017 p. 393 footnote 1). If no objective probabilities are present, as when only considering the unknown Ellsberg two-color urn, then probabilistic sophistication is strictly more general—but then matching probabilities, and our indexes, cannot even be defined.

[6] Under multiple prior theories, this index can be called "perceived level of ambiguity."

Under ambiguity neutrality, with perfect discrimination between single and composite events, $\overline{m_c} = 2/3$ and $\overline{m_s} = 1/3$, and their difference is 1/3. Index a measures how much the actual difference falls short of 1/3. We multiplied by 3 to obtain a convenient normalization with a maximal value 1 (maximal insensitivity, with $\overline{m_c} = \overline{m_s}$).

Ambiguity neutrality gives $a = 0$. We have again calibrated ambiguity neutrality here, controlling for subjective likelihoods by letting them drop from the equations. Empirically, we usually find prevailing insensitivity, $a > 0$, but there are subjects with $a < 0$. For descriptive purposes, it is desirable to allow $a < 0$, which we do. The α-maxmin model, however, does not allow $a < 0$ (Baillon, Li, and Wakker 2018), which is no problem for normative applications that take $a < 0$ to be irrational.

Our two indexes are orthogonal (Baillon, Li, and Wakker 2018). If one is 0, suggesting ambiguity neutrality, the other may still deviate from 0, showing ambiguity attitude. Contrary to what is sometimes suggested in the literature, ambiguity may still play an important role through insensitivity if there is no aversion.[7]

Dimmock et al. (2015) referred to their version of the second index as perceived level of ambiguity. Dimmock et al.'s term, and the multiple priors model underlying it, may serve best for applications that, unlike this paper, have normative aims. However, their assumption of expected utility for risk and their restriction $a \geq 0$ are problematic for descriptive applications. For risk, insensitivity (i.e., inverse-S probability weighting) has been commonly found (Fehr-Duda and Epper 2012; Wakker 2010 §9.5). Our second index naturally extends this insensitivity found under risk to ambiguity, where empirical studies have found that it is usually amplified (Trautmann and van de Kuilen 2015; Wakker 2010 p. 292). Hence, we use the term ambiguity-generated insensitivity (a-insensitivity) to refer to it. Insensitivity was central in the early Einhorn and Hogarth (1985). Gonzalez and Wu (1999) gave an illuminating discussion of its cognitive interpretation, for risk. Wakker (2010, §10.4.2 and §11.8) presents the concept for ambiguity.

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[7] For instance, in Schmeidler's (1989) model, with W denoting the weighting function, $W(E) + W(E^c) = 1$ may hold for all events E, while there may still be strong insensitivity.

3. EXPERIMENT: METHOD

This section presents the experiment. Appendix A gives further details. We investigate the effect of time pressure (TP) on ambiguity. The ambiguity concerns the performance of the AEX (Amsterdam stock exchange) index. Using our method, we can study TP for natural events.

Hypotheses

It is natural to expect that TP will reduce cognitive understanding and, hence, increase the insensitivity index. This is the hypothesis we test. We had no prior prediction about the impact of TP on ambiguity aversion. Ambiguity aversion reflects how much *more* dislike there is for uncertainty than for risk. We saw no reason for this difference to become bigger or smaller.

Subjects

N = 104 subjects participated (56 male, median age 20). They were all students from Erasmus University Rotterdam, recruited from a pool of volunteers. They were randomly allocated to the control and the TP treatment.

The experiment consisted of two parts. Parts 1 and 2 (Table I) each comprised eight questions. They were preceded by a training part (Part 0) of eight questions, to familiarize subjects with the stimuli. All subjects faced the same questions, except that subjects in the TP treatment had to make their choices under time pressure in Part 1. There were 42 subjects in the control treatment and 62 in the TP treatment. The TP sample had more subjects because we expected more variance there.

TABLE I: Organization of the experiment

Within subject / Between subject	*Part 1*	*Part 2*
Control treatment	No time pressure	No time pressure
Time pressure treatment	**Time pressure**	No time pressure

Stimuli: Within- and between-subject treatments

Stimuli: Choice lists

In each question, subjects were asked to choose between two options.

OPTION 1: You win €20 if the AEX index increases/decreases by … between the beginning and the end of the experiment (which lasted 25 minutes on average), and nothing otherwise.

OPTION 2: You win €20 with p% probability and nothing otherwise.

We used choice lists to infer the probability p in Option 2 that leads to indifference between the two options (details in the Appendix). This p is the matching probability of the AEX event. In the TP treatment, a 25-second time limit was set for each choice in Part 1.

Stimuli: Uncertain events

In each part we considered a triple of mutually exclusive and exhaustive single events and their compositions (Table II).

TABLE II: Single AEX-change events for two parts (unit is percentage)[8]

	Event E_1	Event E_2	Event E_3
Part 1	(–100,–0.2)	[–0.2,0.2]	(0.2,∞)
Part 2	(–100,–0.1)	[–0.1,0.3]	(0.3,∞)

We chose the partition in Part 2 somewhat differently to make subjects choose afresh rather than erroneously speculate on relations between the questions. For each part, we measured matching probabilities of all six single and composite events, of which two were repeated to test consistency. The order of the eight questions was randomized for each subject within each part.

[8] In the training Part 0, the events were (–100,–0.4), [–0.4,0.1], and (0.1,∞).

Stimuli: Further questions

At the end of the experiment, subjects were asked to report their age, gender, and nationality.

Incentives

We used the random incentive system. All subjects received a show-up fee of €5 and one of their choices was randomly selected to be played for real (see the Appendix for details).

Analysis

We computed ambiguity aversion and a-insensitivity indexes as explained in §2. Five subjects in the TP treatment did not submit one of their matching probabilities on time and were therefore excluded from the analysis, leaving us with 99 subjects.

We ran OLS regressions to study the impact of TP on a-insensitivity and ambiguity aversion. We clustered standard errors at the individual level because we obtained two values of each index per subject (one for each part). Furthermore, we used Seemingly Unrelated Regressions because the residuals of the regression on a were correlated with the residuals of the regression on b. In the baseline model (Model 1 in the result tables), we took Part 1 in the control treatment as the reference group and considered three dummy variables: part 2*control, part 1*TP and part 2*TP, where each variable takes value 1 if the observation is from the specific part in the specific treatment. We then added control variables (age, gender, and nationality in Model 2) to assess the robustness of the results.

We analyzed responses time to verify that subjects answered faster in the TP treatment. To do so, we ran OLS regressions for the response time with clustered standard errors, as for the indexes. For some events, we elicited the matching probabilities twice to test for consistency, because TP can be expected to decrease consistency. For each treatment and each part, we compared the first and second elicitation of these matching probabilities using t-tests with the Bonferroni correction for multiple comparisons. In the rest of the analysis, we only used the first matching probability elicited for each event.

By set-monotonicity, the matching probability of a composite event should exceed the matching probability of either one of its two constituents. Thus, we tested set-monotonicity six times in each part. *Weak monotonicity* is defined by $\overline{m_c} \geq \overline{m_s}$. It ensures $a \leq 1$. Violations of weak monotonicity entail very erratic answers. We nevertheless kept all answers in the analysis. Excluding the indexes when weak monotonicity was violated did not affect our conclusions (full results in the Online Appendix) unless we report otherwise. We ran non-parametric analysis (Wilcoxon tests and Mann-Whitney U tests) to test whether time pressure had an impact on the number of set-monotonicity and weak monotonicity violations.

4. EXPERIMENT: RESULTS

Table III gives descriptive results for the ambiguity indexes.

TABLE III: Descriptive statistics

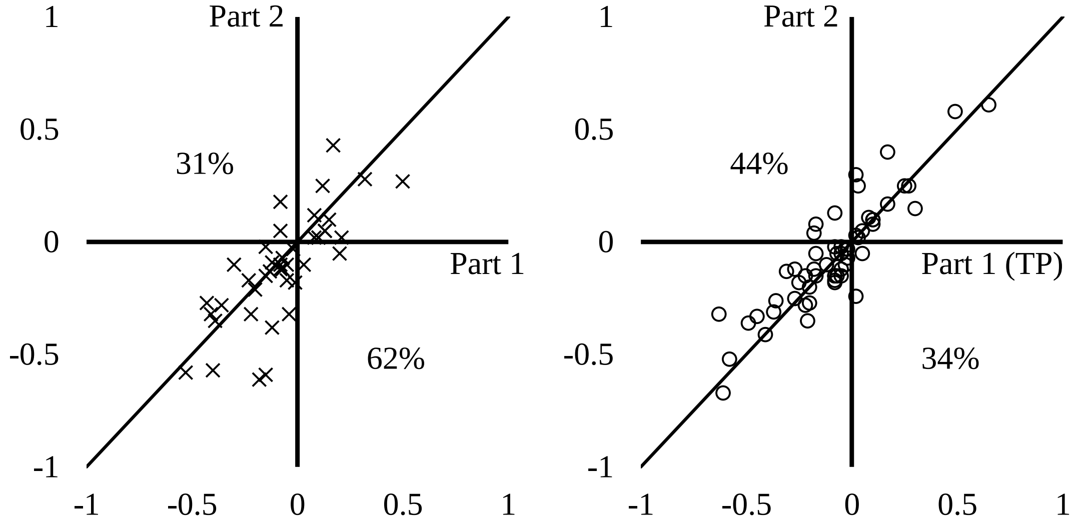
		a (part 1)	a (part 2)	b (part 1)	b (part 2)
Control	*mean*	.15	.17	-.07	-.11
	standard deviation	.44	.41	.21	.24
	standard error	.07	.06	.03	.04
	median	.07	.20	-.08	-.10
	N	42	42	42	42
TP	*mean*	.34	.17	-.09	-.06
	standard deviation	.44	.45	.24	.24
	standard error	.06	.06	.03	.03
	median	.35	.11	-.08	-.11
	N	57	57	57	57

In what follows, we discuss only differences that are significant, with the significance level indicated in the corresponding tables.

4.1. Ambiguity Aversion Index b

To first illustrate the general nature of our data, Figure 1 presents all b indexes of Part 2 as a function of the b indexes of Part 1. Spearman correlations are high (ρ = 0.77 for the control treatment and ρ = 0.85 for the TP treatment) and most dots are in the lower left quadrant or in the upper right quadrant. It shows that subjects are consistently ambiguity averse or consistently ambiguity seeking across parts.

FIGURE 1: Ambiguity aversion indexes *b*



A. Control treatment (ρ = 0.77) B. TP treatment (ρ = 0.85)

Percentages of observations above and below the diagonal are indicated in the figures. Spearman correlations ρ *are in the panel titles.*

Table IV displays the results of the panel regressions for the *b* indexes. In Part 1, the control subjects are slightly ambiguity seeking (–0.07), with the dots in panel A slightly to the left. Regarding our main research question: the null hypothesis that TP has no effect cannot be rejected. The index *b* in TP does not differ significantly from that in the control in Part 1, with dots in panel B not more or less to the left than in panel A. The only effect we find is a learning effect for the control treatment, where Part 2 is a repetition of Part 1.[9] Here ambiguity aversion is lower in Part 2 than in Part 1. There is no learning effect for the TP treatment ($p = 0.14$). This may be because under the TP in Part 1 it was not well possible for subjects to familiarize themselves with the task.

All aforementioned effects, and their levels of significance, are unaffected when controlling for age, gender, and nationality (Dutch / non-Dutch) in Model 2. To test if ambiguity aversion, while not systematically bigger or smaller under TP, would

[9] The learning effect is marginally significant and not significant anymore if we exclude the subjects violating weak monotonicity (Table OB.1 in Online Appendix). To avoid learning effects for the part with TP, we had it precede the part without TP.

become more or less extreme, we test absolute values of *b*, but find no evidence for such effects Online Appendix).

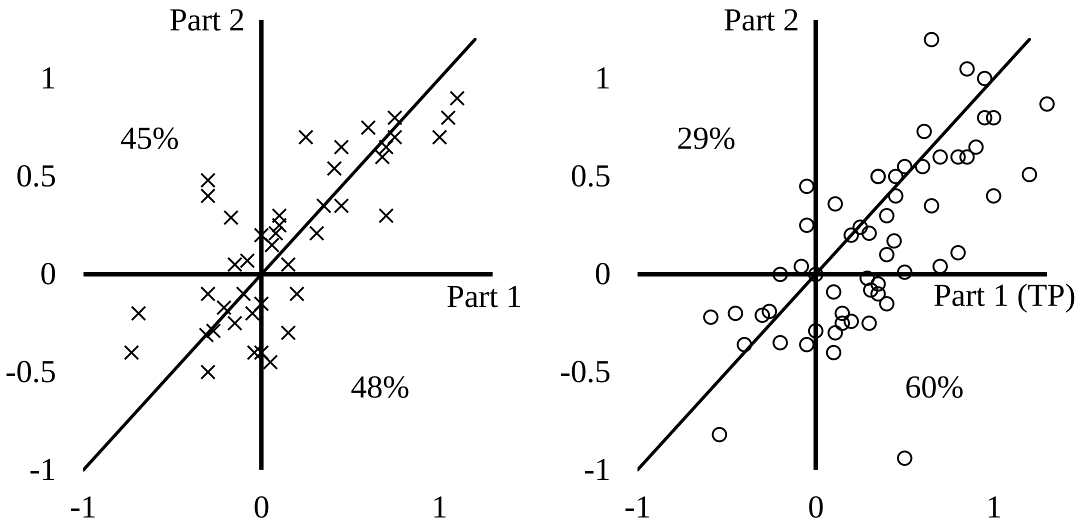
TABLE IV: Ambiguity aversion indexes *b*

	Model 1	Model 2
Intercept	–0.07*	0.02
	(0.03)	(0.06)
Part 1 * TP treatment	**–0.02**	**–0.03**
	(0.05)	**(0.04)**
Part 2 * control treatment	–0.04†	–0.04†
	(0.02)	(0.02)
Part 2 * TP treatment	0.00	–0.01
	(0.05)	(0.04)
Male		–0.08†
		(0.04)
Dutch		–0.07
		(0.05)
Age – 20		0.02
		(0.02)
Chi2	6.42†	18.48**
N	198	198

† $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Point estimates are followed by standard errors between brackets. The impact of TP is in bold. The variable age has been recoded as age − 20 so that the intercept corresponds to the *b* index of a 20-year-old subject (median age).

4.2. A-Insensitivity Index a

FIGURE 2: A-insensitivity indexes a



A. Control treatment (ρ = 0.77) B. TP treatment (ρ = 0.70)

Percentages of observations above and below the diagonal are indicated in the figures. Spearman correlations ρ are in *the panel titles.*

Figure 2 depicts all individual a indexes of Part 2 as a function of the a indexes of Part 1. Spearman correlations are again high (ρ = 0.73 for the control treatment and ρ = 0.74 for TP). Table V displays the results of the panel regressions for the a index. The insensitivity index is between 0.15 and 0.17 for Parts 1 and 2 of the control treatment (no learning effect and points equally split above and below the diagonal in panel A), and also for Part 2 of the TP treatment. However, there is much more a-insensitivity for the TP questions (Part 1 of TP treatment), with $a = 0.34$ and with two-thirds of the dots in panel B to the right of the diagonal. These findings are robust to the addition of control variables (Model 2). Thus, we find a TP effect but no evidence for a learning effect.

TABLE V: A-insensitivity indexes a

	Model 1	Model 2
Intercept	0.15^{*}	$0.20^{\dagger}$
	(0.07)	(0.12)
Part 1 * TP treatment	**0.19^{*}**	**0.18^{*}**
	(0.09)	**(0.09)**
Part 2 * control treatment	0.02	0.02
	(0.04)	(0.04)
Part 2 * TP treatment	0.02	0.01
	(0.09)	(0.09)
Male		–0.05
		(0.08)
Dutch		–0.06
		(0.09)
Age – 20		0.02
		(0.02)
Chi2	16.19^{***}	18.36^{**}
N	198	198

† $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Point estimates are followed by standard errors between brackets. The impact of TP is in bold. The variable age has been recoded as age − 20 so that the intercept corresponds to the a index of a 20-year-old subject (median age).

4.3. *Response Time, Consistency, and Monotonicity*

The average response time in the training part is more than 25 seconds, but this decreases in Part 1 and then again in Part 2 for both the control and the TP treatment. Understandably, subjects needed to familiarize themselves with the task. In Table VI, the benchmark model (Model 1) shows that the average response time of the control subjects in Part 1 is about 17s per matching probability. It is about 4s longer than for subjects under TP, even though subjects could spend up to 25s to answer in the TP-treatment. In Part 2, the control subjects answered faster than in Part 1.

TABLE VI: Response time

	Model 1	Model 2
Intercept	16.63^{***}	16.66^{***}
	(1.33)	(1.44)
Part 1 * TP treatment	**-4.13^{**}**	**-4.44^{**}**
	(1.40)	**(1.44)**
Part 2 * control treatment	-2.33^{*}	-2.33^{*}
	(1.14)	(1.14)
Part 2 * TP treatment	-1.77	-2.08
	(1.74)	(1.79)
Male		-1.45
		(1.21)
Dutch		0.99
		(1.19)
Age – 20		0.48
		(0.32)
R^2	0.02^{**}	0.03^{**}
N	1584	1584

† p < 0.1, * p < 0.05, ** p < 0.01, *** p < 0.001. Point estimates are followed by standard errors between brackets. The impact of TP is in bold. The variable age has been recoded as age − 20 so that the intercept corresponds to the response time of a 20-year-old subject (median age). The stars reported next to R^2 refer to F-tests of overall significance.

We next analyze the consistency of the matching probabilities by comparing repeated elicitations of matching probabilities for some events. Pairwise comparisons for each pair of matching probabilities with the Bonferroni correction indicate one difference, in one of the two tests in Part 1 for the TP treatment: the second matching probability m_{13} is higher than the first one (mean difference = 0.04; p = 0.01). The other differences are not significant.

We find a similar pattern in the set-monotonicity tests. Out of 6 monotonicity tests, the average number of violations is 0.58 in Part 1 for the TP treatment, only 0.30 in Part 2 for the same treatment, and 0.36 and 0.24 in Parts 1 and 2, respectively, for the control treatment. The difference between Parts 1 and 2 in the TP treatment is significant (within-subject Wilcoxon signed-ranks test; Z = −2.61, p = 0.01) and the difference between the TP and the control treatment in Part 1 is marginally significant (between-subject Mann-Whitney U test; Z = −1.71, p = 0.09). The percentage of weak monotonicity violations is 5% and 4% in Parts 1 and 2 for the TP treatment, and 5% and 0% in Parts 1 and 2 for the control treatment. None of the differences is significant.

4.4. *Summary and Discussion of the Experiment*

We summarize the experimental results. TP indeed increases insensitivity (index a), as predicted. That TP harms cognitive understanding is further confirmed by increased violations of consistency and set-monotonicity. These findings confirm Ariely and Zakay's (2001) observation that TP aggravates biases and irrationalities. TP does not increase or decrease ambiguity aversion (index b).

Somewhat similar to our results, Young et al. (2012) found that TP increases insensitivity under risk. The effects of TP on risk aversion are not clear and can go in either direction (Kircher et al. 2017; Kocher, Pahlke, and Trautmann 2013; Young et al. 2012), consistent with our absence of effect on ambiguity aversion. Kocher, Pahlke, and Trautmann (2013) also found increased insensitivity toward outcomes under TP for risk. Tinghög et al. (2013) confirmed a more pronounced four-fold pattern of risk, again in agreement with increased insensitivity. Our second index is therefore useful for future studies and applications regarding TP. Whereas the (ir)rationality of ambiguity aversion has been widely debated, insensitivity clearly reflects cognitive limitation and irrationality. Thus, the increased demand of full insurance and decreased demand for precautionary and partial insurance found by Bajtelsmit, Coats, and Thistle (2015) fits perfectly with insensitivity, as does the decreased quality of decisions in Conte, Scarsini, and Sürücü (2016), De Paola and Gioia (2015), and Kirchler et al. (2017). Our study, therefore, supports the desirability to avoid TP for important decisions.

The absence of ambiguity aversion in our results is not surprising in view of recent studies with similar findings, especially because we used natural events rather than Ellsberg urns (Binmore, Stewart, and Voorhoeve 2012; Charness, Karni, and Levin 2013; Kocher, Lahno, and Trautmann 2017; Trautmann and van de Kuilen 2015). An additional experimental advantage of using natural events—that suspicion about experimenter-manipulated information is avoided—may have contributed to the absence of ambiguity aversion in our study. Such suspicion is further reduced because subjects always bet both on events and on their complements. Finally, the increase in preference (index b) in Part 2 of the control treatment is in agreement with the familiarity bias (Chew, Ebstein, and Zhong 2012; Fox and Levav 2000; Kilka and Weber 2001).

The events in our experiments were natural in the sense that they did not involve any artificially concealed information. We did not consider them in an actually occurring natural decision situation or in a field setting, and the decision situations considered were experimental. However, we used uncertainty that actually occurred and that was relevant to financial traders.

5. GENERAL DISCUSSION

Indexes are simplified summaries of complex realities. Our indexes cannot be expected to perfectly capture ambiguity attitudes in the same way as the well-known index of relative risk aversion (IRR) cannot be expected to perfectly capture risk attitudes for every decision and every theory. The IRR perfectly describes risk attitudes under expected utility with CRRA utility. For other utility functions, it only works well on restricted domains of outcomes (Wakker 2008). Similarly, our indexes perfectly describe ambiguity attitudes under Chateauneuf, Eichberger, and Grant's (2007) neo-additive event weighting for several ambiguity theories (Baillon, Li, and Wakker 2018). In general, they work well if no event in the partition is almost certain or almost impossible. Violations of event additivity and neo-additive weighting occur primarily for extreme events where no theory describes the many irregularities very well.[10]

To date, only few studies have measured ambiguity attitudes for natural events. Many did not control for risk attitudes and could therefore not identify ambiguity attitudes completely (Baillon et al. 2017; Fox, Rogers, and Tversky 1996; Fox and Tversky 1998; Kilka and Weber 2001). Abdellaoui et al. (2011) measured indexes similar to ours but used complex measurements and data fittings, requiring measurements of subjective probabilities, utilities, and event weights. As regards the treatment of unknown beliefs, Brenner and Izhakian (2015) and Gallant, Jahan-Parvar, and Liu (2015) are close to us. They did not assume beliefs given beforehand, but derived them from preferences, as did Abdellaoui et al. (2011). We do not need such

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[10] Thus, for risk, Kahneman and Tversky (1979 pp. 282-283) explicitly refrained from specifying any shape of probability weighting for extreme probabilities.

derivations. Brenner and Izhakian (2015) and Gallant, Jahan-Parvar, and Liu (2015) deviated from our approach in assuming second-order probabilities to capture ambiguity. They made parametric assumptions about the first- and second-order probabilities (assuming normal distributions), including expected utility for risk with constant relative risk aversion, and then fit the remaining parameters to the data for a representative agent. Maccheroni, Marinacci, and Ruffino's (2013) theoretical analysis followed a similar approach. A difficulty of existing parametric fittings is that they are sensitive to the assumptions made about beliefs.

Baillon and Bleichrodt (2015) used a method similarly tractable as ours. They used different indexes[11], and did not establish a control for likelihood. Several papers used indexes similar to those presented above but provided no controls for likelihoods, so that they had to use probability intervals or Ellsberg urns (Baillon, Cabantous, and Wakker 2012; Dimmock t al. 2016a; Dimmock et al. 2015, 2016b). Li (2017), a follow-up of this paper, used our method to study linguistic ambiguities. Such ambiguities are among the most common natural ones. Her sample of Chinese adolescents had an exceptional spread in wealth, allowing for a good measurement of wealth dependence of ambiguity attitudes. Li, Turmunk, and Wakker (2018) used our method to measure the impact of ambiguity attitudes in strategic situations.

This paper showns that the ambiguity aversion index can better be related to matching probabilities than to nonadditive weighting functions W as done before (Dow and Werlang 1992; Schmeidler 1989), to avoid distortions by risk attitudes. An additional advantage of matching probabilities is that they are readily observable. More precisely, we need six indifferences (matching probabilities) to calculate our two indexes. Measuring a nonadditive function W (a theoretical construct: Cozic and Hill 2015) is more difficult, involving other theoretical constructs (U) and theoretical assumptions, which happened in Abdellaoui et al. (2011). In this sense, we make preceding indexes operational (point (d) in Introduction). Because of point (f) in the introduction (validity also if expected utility for risk is violated), our method also works for the general Choquet expected utility model in Gilboa (1987) which, unlike Schmeidler (1989), does not assume expected utility for risk; see Baillon, Li, and

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[11] They used five event-dependent indexes similar to Kilka and Weber (2001), and based on preference conditions of Tversky and Wakker (1995), adapting them to matching probabilities.

Wakker (2018). Key is that risk attitudes, including their deviations, cancel out if we use matching probabilities.

Many studies used introspective likelihood measurements (de Lara Resende and Wu 2010; Fox, Rogers, and Tversky 1996; Fox and Tversky 1998; Ivanov 2011) to capture beliefs for natural events. Professional forecasts and survey data are useful for establishing such beliefs (Anderson, Ghysels, and Juergens 2009). But these are not revealed preference based, and the beliefs may be nonadditive. Then ambiguity attitudes may be captured partially by those nonadditive stated beliefs and partially by their weighting functions and, thus, ambiguity attitudes cannot be clearly isolated. Our paper focuses on clearly defined revealed-preference concepts.

In the popular α-maxmin model (Ghirardato, Maccheroni, and Marinacci 2004), α is often taken as an index of ambiguity aversion. Baillon, Li, and Wakker (2018) show that, for a popular subclass (priors $(1-\varepsilon)Q+\varepsilon T$ with Q a fixed focal probability measure and T variable and any possible probability measure) α can be recovered from our indexes (where our a is their ε):

$$\alpha = \frac{b}{2a} + \frac{1}{2}. \tag{4}$$

Our b can be taken as an index of absolute ambiguity aversion, and α as a relative one, being aversion per perceived unit of ambiguity, renormalized. For readers who prefer the relative index α, our method also shows how to elicit this for natural events with unknown subjective beliefs.

How ambiguity attitudes are related across different sources of uncertainty and across different persons is an important topic for future research. The isolation of ambiguity attitudes from likelihood beliefs provided by this paper will be useful for such research.

6. CONCLUSION

Measuring ambiguity attitudes directly from revealed preferences has up to now only been possible for artificially created uncertainties because no way was known to correct for unknown likelihood beliefs. We introduce a way to control for such beliefs and define two indexes of ambiguity attitudes that apply to natural uncertainties as relevant in applications. This increases external validity. Our indexes are valid for many ambiguity theories, unifying and generalizing several existing indexes. In particular, our indexes are valid if expected utility for risk is violated, which is desirable for empirical purposes. Our second index (insensitivity) captures the likelihood dependence of ambiguity aversion that is usually found empirically.

We apply our indexes in a study on ambiguity under time pressure. Our findings are psychologically plausible, supporting the validity of our indexes: time pressure affects cognitive components (sensitivity/understanding, or level of ambiguity) but not motivational components (ambiguity aversion). Correlations between successive measurements of our indexes are high, confirming the reliability of our method.

APPENDIX A. DETAILS OF THE EXPERIMENT

Procedure

In the experiment, computers of different subjects were separated by wooden panels to minimize interactions between subjects. Brief instructions were read aloud, and tickets with ID numbers were handed out. Subjects typed in their ID numbers to start the experiment. The subjects were randomly allocated to treatment groups through their ID numbers. Talking was not allowed during the experiment. Instructions were given with detailed information about the payment process, user interface, and the type of questions subject would face. The subjects could ask questions to the experimenters at any time. In each session, all subjects started the experiment at the same time.

In the TP treatment, we took two measures to make sure that TP would not have any effect in Parts 0 and 2. First, we imposed a two-minute break after Parts 0 and 1, to avoid spill-over of stress from Part 1 to Part 2. Second, we did not tell the subjects that they would be put under TP prior to Part 1, to avoid stress generated by such an announcement in Part 0 (Ordonez and Benson 1997).

Stimuli: Choice lists

Subjects were asked to state which one of the two choice options in §2 they preferred for different values of p, ascending from 0 to 100 (Figures A.1 and A.2). The midpoint between the two values of p where they switched preference was taken as their indifference point and, hence, as the matching probability.

To help subjects answer the questions quickly, which was crucial under TP, the experimental webpage allowed them to state their preferences with a single click. For example, if they clicked on Option 2 when the probability of winning was 50%, then for all $p > 50\%$, the option boxes for Option 2 were automatically filled out and for all $p < 50\%$ the option boxes for Option 1 were automatically filled out. This procedure also precluded violations of stochastic dominance by preventing multiple preference switches. After clicking on their choices, subjects clicked on a “Submit” button to move to the next question. The response times were also tracked.

In Part 1 of the TP treatment, a timer was displayed showing the time left to answer. If subjects failed to submit their choices before the time limit, their choices would be registered but would not be paid. This happened only 5 out of the 496 times (62 subjects × 8 choices). In a pilot, the average response time without TP was 36 seconds, and another session of the pilot showed that subjects did not experience much TP under a 30-second time limit. Therefore, we chose the 25 seconds limit.

Figure A.1: Screenshot of the experiment software for single event E_3 in Part 0

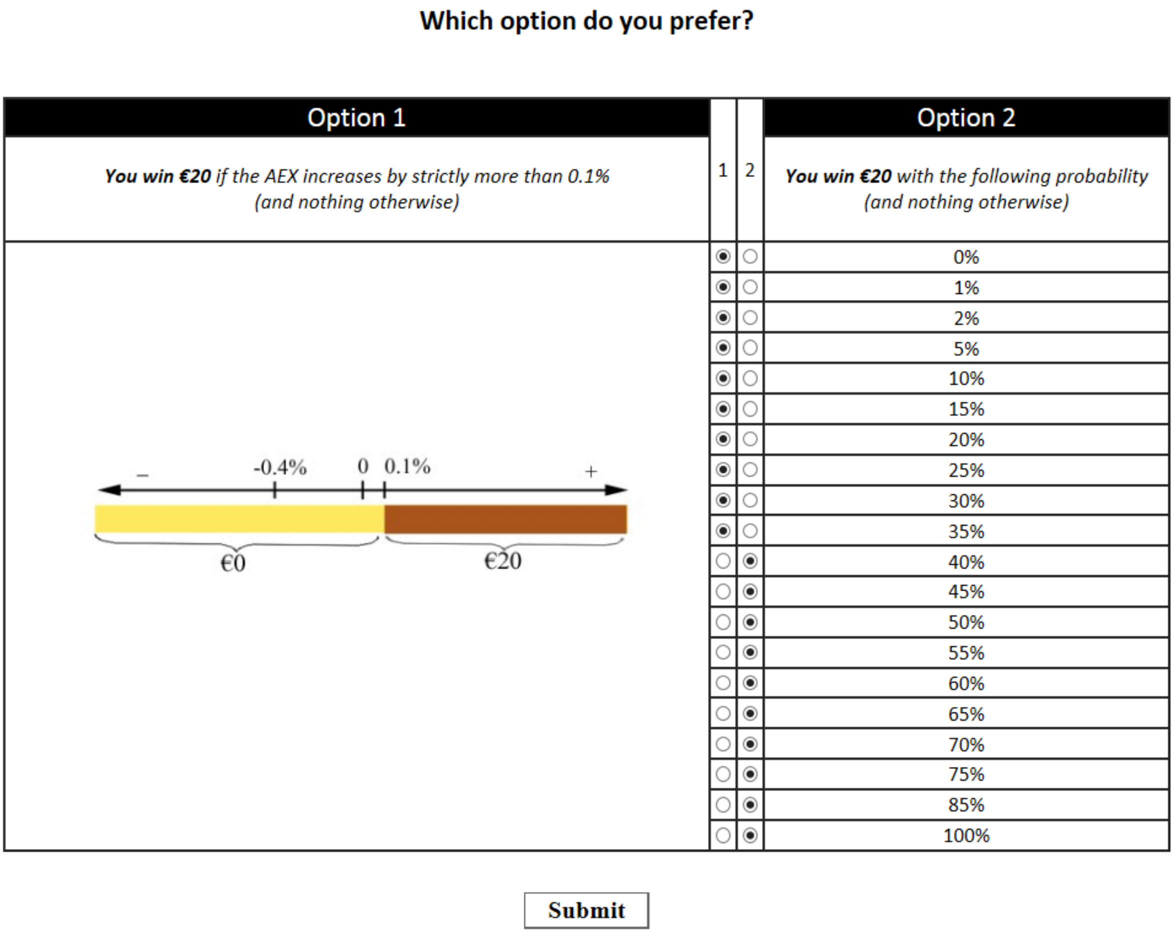
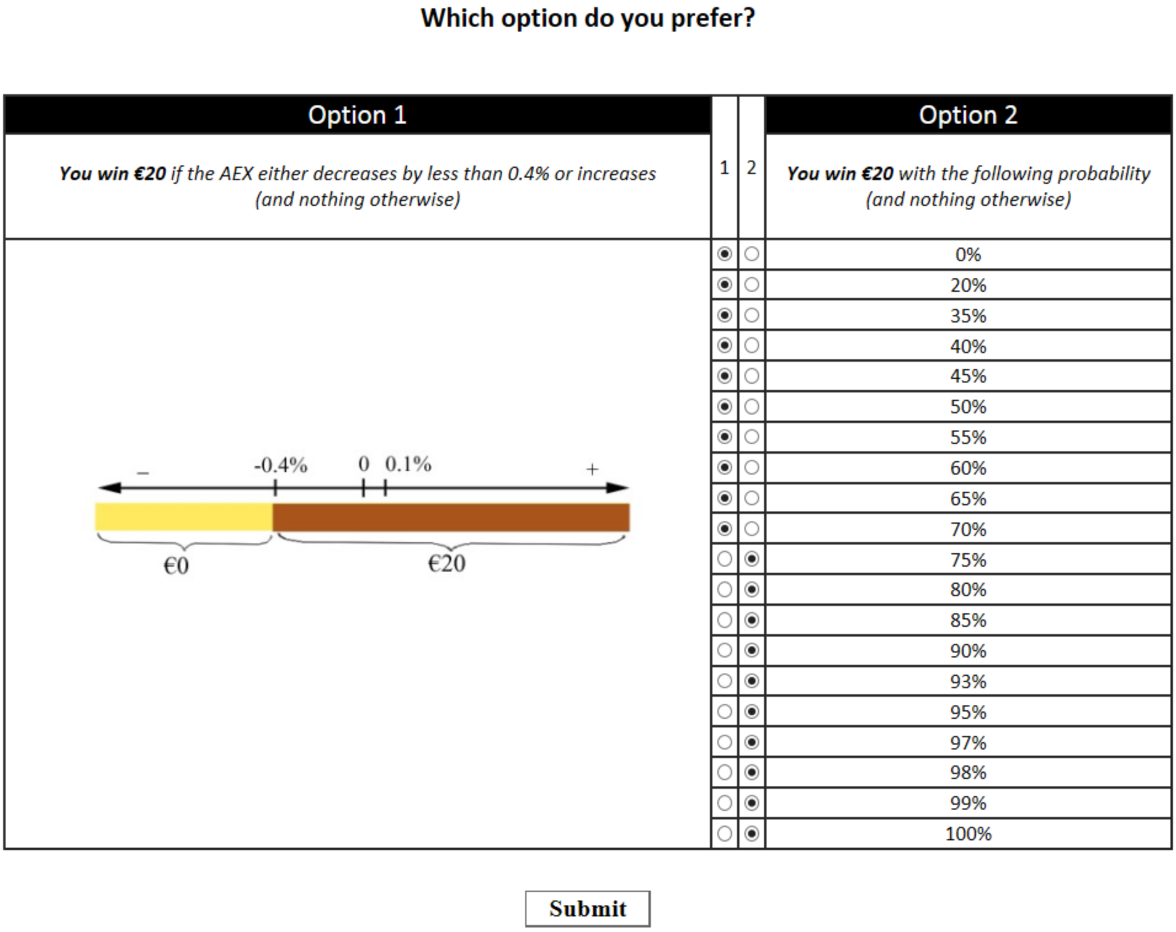


Figure A.2: Screenshot of the experiment software for composite event E_{23} in Part 0



Stimuli: Avoiding middle bias

The middle bias can distort choice lists: subjects tend to choose the options, in our case the preference switch, that are located in the middle of the provided range (Erev and Ert 2013; Poulton 1989). TP can be expected to reinforce this bias. If we had used a common equally-spaced choice list with, for example, 5% incremental steps, then the middle bias would have moved matching probabilities in the direction of 50% (both for the single and composite events). This bias would have enhanced the main phenomenon found in this paper, a-insensitivity, and rendered our findings less convincing. To avoid this problem, we designed choice lists that were not equally spaced. In our design, the middle bias enhanced matching probabilities 0.35 for single events and probabilities 0.70 for composite events. Thus, this bias enhanced additivity of the matching probabilities, decreased a-insensitivity, and moved our a-insensitivity index toward 0. It makes findings of nonadditivity and a-insensitivity more convincing.

Table A.1 lists the AEX events that we used. Some questions were repeated for consistency checks. The corresponding events are listed twice.

TABLE A.1: List of events on which the AEX prospects were based

Part	**Event**	**Event description**
0 (Training)	E_1	the AEX decreases by strictly more than 0.4%
	E_1	the AEX decreases by strictly more than 0.4%
	E_2	the AEX either decreases by less than 0.4% or increases by less than 0.1%
	E_3	the AEX increases by strictly more than 0.1%
	E_{12}	the AEX either increases by less than 0.1% or decreases
	E_{23}	the AEX either decreases by less than 0.4% or increases
	E_{23}	the AEX either decreases by less than 0.4% or increases
	E_{13}	the AEX either decreases by strictly more than 0.4% or increases by strictly more than 0.1%
1	E_1	the AEX decreases by strictly more than 0.2%
	E_2	the AEX either decreases by less than 0.2% or increases by less than 0.2%
	E_2	the AEX either decreases by less than 0.2% or increases by less than 0.2%
	E_3	the AEX increases by strictly more than 0.2%
	E_{12}	the AEX either increases by less than 0.2% or decreases
	E_{12}	the AEX either increases by less than 0.2% or decreases
	E_{23}	the AEX either decreases by less than 0.2% or increases
	E_{13}	the AEX either decreases by strictly more than 0.2% or increases by strictly more than 0.2%
2	E_1	the AEX decreases by strictly more than 0.1%
	E_2	the AEX either decreases by less than 0.1% or increases by less than 0.3%
	E_3	the AEX increases by strictly more than 0.3%
	E_3	the AEX increases by strictly more than 0.3%
	E_{12}	the AEX either increases by less than 0.3% or decreases
	E_{23}	the AEX either decreases by less than 0.1% or increases
	E_{13}	the AEX either decreases by strictly more than 0.1% or increases by strictly more than 0.3%
	E_{13}	the AEX either decreases by strictly more than 0.1% or increases by strictly more than 0.3%

Incentives

For each subject, one question (i.e., one row of one choice list) was randomly selected to be played for real at the end of the experiment. If subjects preferred the bet on the stock market index, then the outcome was paid according to the change in the stock market index during the experiment. Bets on the given probabilities were settled using dice. In the instructions of the experiment, subjects were presented with two examples to familiarize them with the payment scheme. If the time deadline for a TP question was not met, the worst outcome (no payoff) resulted. Therefore, it was in the subjects' interest to submit their choices on time.

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REFERENCES

Abdellaoui, Mohammed, Aurélien Baillon, Laetitia Placido, and Peter P. Wakker (2011) "The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation," *American Economic Review* 101, 695–723.

Anderson, Evan W., Eric Ghysels, and Jennifer L. Juergens (2009) "The Impact of Risk and Uncertainty on Expected Returns," *Journal of Financial Economics* 94, 233–263.

Ariely, Dan and Dan Zakay (2001) "A Timely Account of the Role of Duration in Decision Making," *Acta Psychologica* 108, 187–207.

Baillon, Aurélien and Han Bleichrodt (2015) "Testing Ambiguity Models through the Measurement of Probabilities for Gains and Losses," *American Economic Journal: Microeconomics* 7, 77–100.

Baillon, Aurélien, Han Bleichrodt, Umut Keskin, Olivier L'Haridon, and Chen Li (2017) "Learning under Ambiguity: An Experiment Using Initial Public Offerings on a Stock Market," *Management Science,* forthcoming.

Baillon, Aurélien, Laure Cabantous, and Peter P. Wakker (2012) "Aggregating Imprecise or Conflicting Beliefs: An Experimental Investigation Using Modern Ambiguity Theories," *Journal of Risk and Uncertainty* 44, 115–147.

Baillon, Aurélien, Chen Li, and Peter P. Wakker (2018) "Balanced Design: The Key to Measuring Ambiguity Attitudes when Beliefs Are Unknown," working paper.

Bajtelsmit, Vickie, Jennifer C. Coats, and Paul Thistle (2015) "The Effect of Ambiguity on Risk Management Choices: An Experimental Study," *Journal of Risk and Uncertainty* 50:249–280.

Binmore, Ken, Lisa Stewart, and Alex Voorhoeve (2012) "How Much Ambiguity Aversion? Finding Indifferences between Ellsberg's Risky and Ambiguous Bets," *Journal of Risk and Uncertainty* 45, 215–238.

Bleichrodt, Han, José Luis Pinto, and Peter P. Wakker (2001) "Making Descriptive Use of Prospect Theory to Improve the Prescriptive Use of Expected Utility," *Management Science* 47, 1498–1514.

Brenner, Menachem and Yehuda Izhakian (2015) "Asset Pricing and Ambiguity: Empirical Evidence," working paper.

Budescu, David V., Han-Hui Por, Stephen B. Broomell, and Michael Smithson (2014) "The Interpretation of IPCC Probabilistic Statements around the World," *Nature Climate Change* 4, 508–512.

Callen, Michael, Mohammed Isaqzadeh, James D. Long, and Charles Sprenger (2013) "Violence and Risk Preferences: Artefactual and Experimental Evidence from Afghanistan," *American Economic Review* 104, 123–148.

Camerer, Colin F. and Martin Weber (1992) "Recent Developments in Modeling Preferences: Uncertainty and Ambiguity," *Journal of Risk and Uncertainty* 5, 325–370.

Charness, Gary, Edi Karni, and Dan Levin (2013) "Ambiguity Attitudes and Social Interactions: An Experimental Investigation," *Journal of Risk and Uncertainty* 46, 1–25.

Chateauneuf, Alain and José H. Faro (2009) "Ambiguity through Confidence Functions," *Journal of Mathematical Economics* 45, 535–558.

Chateauneuf, Alain, Jürgen Eichberger, and Simon Grant (2007) "Choice under Uncertainty with the Best and Worst in Mind: NEO-Additive Capacities," *Journal of Economic Theory* 137, 538–567.

Chew, Soo Hong, Richard P. Ebstein, and Songfa Zhong (2012) "Ambiguity Aversion and Familiarity Bias: Evidence from Behavioral and Gene Association Studies," *Journal of Risk and Uncertainty* 44, 1–18.

Chew, Soo Hong, King King Li, Robin Chark, and Songfa Zhong (2008) "Source Preference and Ambiguity Aversion: Models and Evidence from Behavioral and Neuroimaging Experiments." *In* Daniel Houser and Kevin McGabe (eds.) *Neuroeconomics. Advances in Health Economics and Health Services Research* 20, 179–201, JAI Press, Bingley, UK.

Cohen, Michèle and Jean-Yves Jaffray (1980) "Rational Behavior under Complete Ignorance," *Econometrica* 48, 1281–1299.

Conte, Anna, Marco Scarsini, and Oktay Sürücü (2016) "The Impact of Time Limitation: Insights from a Queueing Experiment," *Journal of Behavioral Decision Making* 11, 260–274.

de Lara Resende, José G., and George Wu (2010) "Competence Effects for Choices Involving Gains and Losses," *Journal of Risk and Uncertainty* 40, 109–132.

De Paola, Maria and Francesca Gioia (2015) "Who Performs Better under Time Pressure? Results from a Field Experiment," *Journal of Experimental Psychology* 53, 37–53.

Dean, Mark and Pietro Ortoleva (2017) "Allais, Ellsberg, and Preferences for Hedging," *Theoretical Economics* 12, 377–424.

Dimmock, Stephen G., Roy Kouwenberg, Olivia S. Mitchell, and Kim Peijnenburg (2015) "Estimating Ambiguity Preferences and Perceptions in Multiple Prior Models: Evidence from the Field," *Journal of Risk and Uncertainty* 51 219–244.

Dimmock, Stephen G., Roy Kouwenberg, and Peter P. Wakker (2016a) "Ambiguity Attitudes in a Large Representative Sample," *Management Science* 62, 1363–1380.

Dimmock, Stephen G., Roy Kouwenberg, Olivia S. Mitchell, and Kim Peijnenburg (2016b) "Ambiguity Aversion and Household Portfolio Choice Puzzles: Empirical Evidence," *Journal of Financial Economics*119, 559–577.

Dobbs, Ian M. (1991) "A Bayesian Approach to Decision-Making under Ambiguity," *Economica* 58, 417–440.

Dow, James and Sérgio R.C. Werlang (1992) "Uncertainty Aversion, Risk Aversion and the Optimal Choice of Portfolio," *Econometrica* 60, 197–204.

Easley, David and Maureen O'Hara (2009) "Ambiguity and Nonparticipation: The Role of Regulation," *Review of Financial Studies* 22, 1817–1843.

Einhorn, Hillel J. and Robin M. Hogarth (1985) "Ambiguity and Uncertainty in Probabilistic Inference," *Psychological Review* 92, 433–461.

Ellsberg, Daniel (1961) "Risk, Ambiguity and the Savage Axioms," *Quarterly Journal of Economics* 75, 643–669.

Ellsberg, Daniel (2011) "Notes on the Origins of the Ellsberg Urns (Introduction to the Symposium Issue)," *Economic Theory* 48, 221–227.

Erev, Ido and Eyal Ert (2013), "On the Descriptive Value of Loss Aversion in Decisions under Risk," *Judgment and Decision Making* 8, 214–235.

Essl, Andrea and Stefanie Jaussi (2017) "Choking under Time Pressure: The Influence of Deadline-Dependent Bonus and Malus Incentive Schemes on Performance," *Journal of Economic Behavior and Organization* 133, 127–137.

Etner, Johanna, Meglena Jeleva, and Jean-Marc Tallon (2012) "Decision Theory under Ambiguity," *Journal of Economic Surveys* 26, 234–270.

Fehr-Duda, Helga and Thomas Epper (2012) "Probability and Risk: Foundations and Economic Implications of Probability-Dependent Risk Preferences," *Annual Review of Economics* 4, 567–593.

Fox, Craig R. and Jonathan Levav (2000) "Familiarity Bias and Belief Reversal in Relative Likelihood Judgment," *Organizational Behavior and Human Decision Processes* 82, 268–292.

Fox, Craig R., Brett A. Rogers, and Amos Tversky (1996) "Options Traders Exhibit Subadditive Decision Weights," *Journal of Risk and Uncertainty* 13, 5–17.

Fox, Craig R. and Amos Tversky (1998) "A Belief-Based Account of Decision under Uncertainty," *Management Science* 44, 879–895.

Gallant, A. Ronald, Mohammad R. Jahan-Parvar, and Hening Liu (2015) "Measuring Ambiguity Aversion," working paper.

Gayer, Gabi (2010) "Perception of Probabilities in Situations of Risk; A Case Based Approach," *Games and Economic Behavior* 68, 130–143.

Ghirardato, Paolo, Fabio Maccheroni, and Massimo Marinacci (2004) "Differentiating Ambiguity and Ambiguity Attitude," *Journal of Economic Theory* 118, 133–173.

Ghirardato, Paolo and Massimo Marinacci (2002) "Ambiguity Made Precise: A Comparative Foundation," *Journal of Economic Theory* 102, 251–289.

Gilboa, Itzhak (1987) "Expected Utility with Purely Subjective Non-Additive Probabilities," *Journal of Mathematical Economics* 16, 65–88.

Gilboa, Itzhak and Massimo Marinacci (2016) "Ambiguity and the Bayesian Paradigm." In Horacio Arló-Costa, Vincent F. Hendricks, and Johan F.A.K. van Benthem (eds.), *Readings in Formal Epistemology*, 385–439. Springer, Berlin.

Gilboa, Itzhak and David Schmeidler (1989) "Maxmin Expected Utility with a Non-Unique Prior," *Journal of Mathematical Economics* 18, 141–153.

Gonzalez, Richard and George Wu (1999) "On the Shape of the Probability Weighting Function," *Cognitive Psychology* 38, 129–166.

Guidolin, Massimo and Francesca Rinaldi (2013) "Ambiguity in Asset Pricing and Portfolio Choice: A Review of the Literature," *Theory and Decision* 74, 183–217.

Gul, Faruk and Wolfgang Pesendorfer (2014) "Expected Uncertain Utility Theory," *Econometrica* 82, 1–39.

Halter, Alfred N. and Christopher Beringer (1960) "Cardinal Utility Functions and Managerial Behavior," *Journal of Farm Economics* 42, 118–132.

Heath, Chip and Amos Tversky (1991) "Preference and Belief: Ambiguity and Competence in Choice under Uncertainty," *Journal of Risk and Uncertainty* 4, 5–28.

Cozic, Mikael and Brian Hill (2015) "Representation Theorems and the Semantics of Decision-Theoretic Concepts," *Journal of Economic Methodology* 22, 292–311.

Ivanov, Asen (2011) "Attitudes to Ambiguity in One-Shot Normal-Form Games: An Experimental Study," *Games and Economic Behavior* 71, 366–394.

Izhakian, Yehuda (2017) "Expected Utility with Uncertain Probabilities Theory," *Journal of Mathematical Economics* 69, 91–103.

Jaffray, Jean-Yves (1994) "Dynamic Decision Making with Belief Functions." *In* Ronald R. Yager, Mario Fedrizzi, and Janus Kacprzyk (eds.) *Advances in the Dempster-Shafer Theory of Evidence*, 331–352, Wiley, New York.

Kahn, Barbara E. and Rakesh K. Sarin (1988) "Modeling Ambiguity in Decisions under Uncertainty," *Journal of Consumer Research* 15, 265–272.

Kahneman, Daniel and Amos Tversky (1979) "Prospect Theory: An Analysis of Decision under Risk," *Econometrica* 47, 263–291.

Keynes, John Maynard (1921) "*A Treatise on Probability.*" McMillan, London. 2nd edn. 1948.

Kilka, Michael and Martin Weber (2001) "What Determines the Shape of the Probability Weighting Function under Uncertainty," *Management Science* 47, 1712–1726.

Kirchler, Michael, David Andersson, Caroline Bonn, Magnus Johannesson, Erik Ø. Sørensen, Matthias Stefan, Gustav Tinghög, and Daniel Västfjäll (2017) "The

Effect of Fast and Slow Decisions on Risk Taking," *Journal of Risk and Uncertainty* 54, 37–59.

Klibanoff, Peter, Massimo Marinacci, and Sujoy Mukerji (2005) "A Smooth Model of Decision Making under Ambiguity," *Econometrica* 73, 1849–1892.

Knight, Frank H. (1921) "*Risk, Uncertainty, and Profit."* Houghton Mifflin, New York.

Kocher, Martin G., Amrei Marie Lahno, and Stefan T. Trautmann (2017) "Ambiguity Aversion is not Universal," *European Economic Review*, forthcoming.

Kocher, Martin G., Julius Pahlke, and Stefan T. Trautmann (2013) "Tempus Fugit: Time Pressure in Risky Decisions," *Management Science* 59, 2380–2391.

Lehrer, Ehud and Roee Teper (2015) "Subjective Independence and Concave Expected Utility," *Journal of Economic Theory* 158, 33–53.

Li, Chen (2017) "Are the Poor Worse at Dealing with Ambiguity? Ambiguity Attitude of Urban and Rural Chinese Adolescents," *Journal of Risk and Uncertainty* 54, 239–268.

Li, Chen, Uyanga Turmunkh, and Peter P. Wakker (2018) "Trust as a Decision under Ambiguity," *Experimental Economics*, forthcoming.

Maccheroni, Fabio, Massimo Marinacci, and Doriana Ruffino (2013) "Alpha as Ambiguity: Robust Mean-Variance Portfolio Analysis," *Econometrica* 81, 1075–1113.

Maccheroni, Fabio, Massimo Marinacci, and Aldo Rustichini (2006) "Ambiguity Aversion, Robustness, and the Variational Representation of Preferences," *Econometrica* 74, 1447–1498.

Machina, Mark J. and Marciano Siniscalchi (2014) "Ambiguity and Ambiguity Aversion." *In* Mark J. Machina and W. Kip Viscusi (Eds.) "*Handbook of the Economics of Risk and Uncertainty*, Vol. 1," North-Holland, Amsterdam.

Marinacci, Massimo (2015) "Model Uncertainty," *Journal of the European Economic Association* 13, 1022–1100.

Nau, Robert F. (2006) "Uncertainty Aversion with Second-Order Utilities and Probabilities," *Management Science* 52, 136–145.

Neilson, William S. (2010) "A Simplified Axiomatic Approach to Ambiguity Aversion," *Journal of Risk and Uncertainty* 41, 113–124.

Olszewski, Wojciech (2007) "Preferences over Sets of Lotteries," *Review of Economic Studies* 74, 567–595.

Ordonez, Lisa and Lehman Benson (1997) "Decisions under Time Pressure: How Time Constraint Affects Risky Decision Making," *Organizational Behavior and Human Decision Processes* 71, 121–140.

Poulton, E. Christopher (1989) "*Bias in Quantifying Judgments."* Erlbaum, Hillsdale NJ.

Pratt, John W. (1964) "Risk Aversion in the Small and in the Large," *Econometrica* 32, 122–136.

Schmeidler, David (1989) "Subjective Probability and Expected Utility without Additivity," *Econometrica* 57, 571–587.

Shaw, W. Douglass (2016) "Environmental and Natural Resource Economics Decisions under Risk and Uncertainty: A Survey," *International Review of Environmental and Resource Economics* 9, 1–130.

Siniscalchi, Marciano (2009) "Vector Expected Utility and Attitudes toward Variation," *Econometrica* 77, 801–855.

Spiliopoulos, Leonidas and Andreas Ortmann (2017) "The BCD of Response Time Analysis in Experimental Economics," *Experimental Economics*, forthcoming.

Tinghög, Gustav, David Andersson, Caroline Bonn, Harald Böttiger, Camilla Josephson, Gustaf Lundgren, Daniel Västfjäll, Michael Kirchler, Magnus Johannesson (2013) "Intuition and Cooperation Reconsidered," *Nature* 498 (06 June 2013), pp. E1–E2.

Trautmann, Stefan T. and Gijs van de Kuilen (2015) "Ambiguity Attitudes." *In* Gideon Keren and George Wu (eds.), *The Wiley Blackwell Handbook of Judgment and Decision Making*, 89–116, Blackwell, Oxford, UK.

Tversky, Amos and Daniel Kahneman (1992) "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty* 5, 297–323.

Tversky, Amos and Peter P. Wakker (1995) "Risk Attitudes and Decision Weights," *Econometrica* 63, 1255–1280.

Viscusi, W. Kip and Wesley A. Magat (1992) "Bayesian Decisions with Ambiguous Belief Aversion," *Journal of Risk and Uncertainty* 5, 371–387.

Wakker, Peter P. (2008) "Explaining the Characteristics of the Power (CRRA) Utility Family," *Health Economics* 17, 1329–1344.

Wakker, Peter P. (2010) "*Prospect Theory: For Risk and Ambiguity*." Cambridge University Press, Cambridge, UK.

Young, Diana L., Adam S. Goodie, Daniel B. Hall, and Eric Wu (2012) "Decision Making under Time Pressure, Modeled in a Prospect Theory Framework," *Organizational Behavior and Human Decision Processes* 118, 179–188.