Online appendix of "Prudence with respect to ambiguity"

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The first section recalls the definitions of risk apportionment. Section 2 introduces notation and section 3 defines ambiguity apportionment. Section 4 gives results for second-order expected utility and multiplier preferences and in section 5, a way to adapt the definitions to ambiguity models with non-expected utility under risk is proposed. Finally, section 6 presents simple models of precautionary saving with ambiguity.

1 Risk apportionment

Let us recall the definitions of Eeckhoudt and Schlesinger (2006, henceforth *E&S*) with their notation. Preferences are *monotone* if $B_1^{risk} = 0$ is preferred to $A_1^{risk} = -\kappa$ for all $-\kappa < 0$ and all initial wealth levels, and *risk averse* if $B_2^{risk} = 0$ is preferred to $A_2^{risk} = x_0$ for all zero-mean lottery x_0 and all initial wealth levels. The decision maker's preferences satisfy *risk prudence* if $B_3^{risk} = (-\kappa)_{\frac{1}{2}} x_0$ is preferred to $A_3^{risk} = 0_{\frac{1}{2}}(x_0 - \kappa)$ for all $-\kappa < 0$, all zero-mean lottery x_0, and all initial wealth levels. Her preferences satisfy *risk temperance* if $B_4^{risk} = x_0^1{}_{\frac{1}{2}} x_0^2$ is preferred to $A_4^{risk} = 0_{\frac{1}{2}}(x_0^2 + x_0^1)$ for all independent zero-mean lottery x_0^1 and x_0^2 and all initial wealth levels. Like risk prudence, risk temperance signals a distaste for two simultaneous "harms" (two zero-mean risks).

Risk prudence and risk temperance are risk apportionments of order 3 and 4, respectively. The 5^{th}-order, *risk edginess*, involves a 50-50 lottery yielding B_3^{risk} or A_3^{risk} and the obligation to assign an extra zero-mean (independent) lottery to one of them. Risk edginess holds if $B_5^{risk} = A_3^{risk}{}_{\frac{1}{2}}(x_0^2 + B_3^{risk})$ is preferred to $A_5^{risk} = B_3^{risk}{}_{\frac{1}{2}}(x_0^2 + A_3^{risk})$ for all

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independent zero-mean lottery x_0^1 and x_0^2 and all initial wealth levels. The generalizations for any order n follow the same logic. Let $n \setminus m$ be the *integer division*, i.e., the integer part of $\frac{n}{m}$.

Definition 1. *The decision maker exhibits risk apportionment of order n if $B_n^{risk} = (0 + A_{n-2}^{risk})_{\frac{1}{2}}(x_0^{n\setminus 2} + B_{n-2}^{risk})$ is preferred to $A_n^{risk} = (0 + B_{n-2}^{risk})_{\frac{1}{2}}(x_0^{n\setminus 2} + A_{n-2}^{risk})$ for all initial wealth levels, all $-\kappa < 0$, and all sets of mutually-independent zero-mean lottery x_0^j.*

Risk apportionment determines the sign of the successive derivatives of the utility function in an expected utility framework:

Theorem (E&S). *Let n be any strictly positive natural number. Assume expected utility with u n-times differentiable. Then, the following two statements are equivalent:*

(i) Risk apportionment of order n.

(ii) $sgn\left(u^{(n)}\right) = (-1)^{n+1}$.

2 Notation

2.1 Matrices of events and lotteries

In the present online-appendix, we will use matrices to generalise the definitions proposed in the main text. For instance, in the definition of ambiguity aversion, we only considered one binary spread in probability ($\pm\varepsilon$) and, therefore, only two events. To allow for a series of t binary spreads, we will need $2 \times t$ events, which we will represent in a $2 \times t$ matrix. The t binary spreads in probabilities will also be represented in a $2 \times t$ matrix. The matrix $[E_{i,j}]_{m\times t}$ refers to an $(m \times t)$-fold *partition* of S. In the following, t will be neglected for column vectors ($t = 1$). We will assign to the events of a matrix various mixtures of the same two lotteries. We denote *matrices of mixtures* of two lotteries x and y by $x_{[\alpha_{i,j}]_{m\times t}}y$. With this notation, it is implicitly assumed that all $\alpha_{i,j}$s belong to $[0, 1]$. Finally, $\langle [E_{i,j}]_{m\times t}, x_{[\alpha_{i,j}]_{m\times t}}y\rangle$ refers to the act assigning the lottery $x_{\alpha_{i,j}}y$ (which is a mixture of x and y) to all $s \in E_{i,j}$.

We can use this notation to describe the traditional Ellsberg experiment. In this experiment, the decision maker wins \$10 if she draws a red ball from an urn, and she can choose between two possible urns: a known urn that contains 50 black balls and 50 red balls (Option B) or an ambiguous urn that contains 100 balls, red or black in unknown proportion (Option A). Denoting E_i as the event "there are i red balls in the ambiguous urn", the choice can be represented as follows:

$$\text{Option A} = \left\langle \begin{bmatrix} E_0 & \cdots & E_{49} & E_{50} \\ E_{100} & \cdots & E_{51} & \varnothing \end{bmatrix}, 10 \begin{bmatrix} 0 & \cdots & 0.49 & 0.5 \\ 1 & \cdots & 0.51 & 0.5 \end{bmatrix} 0 \right\rangle \tag{1}$$

$$\text{vs. Option B} = \left\langle \begin{bmatrix} E_0 & \cdots & E_{49} & E_{50} \\ E_{100} & \cdots & E_{51} & \varnothing \end{bmatrix}, 10 \begin{bmatrix} 0.5 & \cdots & 0.5 & 0.5 \\ 0.5 & \cdots & 0.5 & 0.5 \end{bmatrix} 0 \right\rangle. \tag{2}$$

The decision maker is then asked which urn she would prefer if she could win when drawing a black ball. Thus, she can now choose between

$$\text{Option A'} = \left\langle \begin{bmatrix} E_{100} & \cdots & E_{51} & \varnothing \\ E_0 & \cdots & E_{49} & E_{50} \end{bmatrix}, 10 \begin{bmatrix} 0 & \cdots & 0.49 & 0.5 \\ 1 & \cdots & 0.51 & 0.5 \end{bmatrix} 0 \right\rangle \tag{3}$$

$$\text{vs. Option B'} = \left\langle \begin{bmatrix} E_{100} & \cdots & E_{51} & \varnothing \\ E_0 & \cdots & E_{49} & E_{50} \end{bmatrix}, 10 \begin{bmatrix} 0.5 & \cdots & 0.5 & 0.5 \\ 0.5 & \cdots & 0.5 & 0.5 \end{bmatrix} 0 \right\rangle. \tag{4}$$

Option A' and Option B' are derived from Options A and B by permuting the two rows of the matrices of events. For prudence and higher orders, we will have to permute more than two rows. The following section introduces such permutations

2.2 Permutations of matrices of events

In definition 1 (ambiguity aversion), we permuted the two outcomes (ε, $-\varepsilon$) over the two events (E, E^c). With series of binary spreads (ε_j, $-\varepsilon_j$) organised in a $2 \times t$ matrix, the equivalent will be to permute entire rows, as shown in the Ellsberg example above. We therefore consider the two *symmetric permutations* of a matrix $[E_{i,j}]_{2\times t}$:

$$\begin{bmatrix} E_{1,1} & \dots & E_{1,t} \\ E_{2,1} & \dots & E_{2,t} \end{bmatrix} \text{ and } \begin{bmatrix} E_{2,1} & \dots & E_{2,t} \\ E_{1,1} & \dots & E_{1,t} \end{bmatrix}. \tag{5}$$

For a $[E_{i,j}]_{4\times t}$ matrix, we consider the following four *symmetric permutations*:

$$\begin{bmatrix} E_{1,1} & \dots & E_{1,t} \\ E_{2,1} & \dots & E_{2,t} \\ E_{3,1} & \dots & E_{3,t} \\ E_{4,1} & \dots & E_{4,t} \end{bmatrix} \text{ and } \begin{bmatrix} E_{2,1} & \dots & E_{2,t} \\ E_{1,1} & \dots & E_{1,t} \\ E_{4,1} & \dots & E_{4,t} \\ E_{3,1} & \dots & E_{3,t} \end{bmatrix} \text{ and } \begin{bmatrix} E_{3,1} & \dots & E_{3,t} \\ E_{4,1} & \dots & E_{4,t} \\ E_{1,1} & \dots & E_{1,t} \\ E_{2,1} & \dots & E_{2,t} \end{bmatrix} \text{ and } \begin{bmatrix} E_{4,1} & \dots & E_{4,t} \\ E_{3,1} & \dots & E_{3,t} \\ E_{2,1} & \dots & E_{2,t} \\ E_{1,1} & \dots & E_{1,t} \end{bmatrix}. \tag{6}$$

The same logic can be applied to any $2^n \times t$ matrix. Let us first define, for a matrix $[E_{i,j}]_{2^n\times t}$, a *permutation of length* 2^m, which permutes $E_{h2^m+i,j}$ with $E_{(h+1)2^m+i,j}$ for all $j \in \{1, \cdots, t\}$, all $i \in \{1, \cdots, 2^m\}$, and all $h \in \{0, 2, \cdots, 2^{n-m} - 2\}$. For a matrix of size $2^n \times t$, we consider the n permutations of size 2^m for $0 \leq m < n$ (thus including no permutation) and all possible combinations[1] of them. This gives us 2^n *symmetric permutations* of $[E_{i,j}]_{2^n\times t}$ because the number of k-combinations of n elements for all k such that $0 \leq k \leq n$ is 2^n. The function $\boldsymbol{\sigma}(\cdot)$ will denote a generic symmetric permutation. This construction of symmetric permutations ensures that no event is in the same place twice when considering all 2^n symmetric permutations of a matrix of events.

Using these permutations, we can say that *there is a symmetric permutation of* $[E_{i,j}]_{m\times t}$ *such that* $x_{[\beta_{i,j}]_{m\times t}}y$ *is preferred to* $x_{[\alpha_{i,j}]_{m\times t}}y$ if there exists a symmetric permutation $\boldsymbol{\sigma}$ such that $\langle \boldsymbol{\sigma}\left([E_{i,j}]_{m\times t}\right), x_{[\beta_{i,j}]_{m\times t}}y\rangle \succsim \langle \boldsymbol{\sigma}\left([E_{i,j}]_{m\times t}\right), x_{[\alpha_{i,j}]_{m\times t}}y\rangle$.

2.3 Operations on matrices of mixtures

Two (rather unusual) operations are used for matrices of mixtures. The first one, $\cup$ (also denoted $\bigcup$), is the (vertical) *union* of two matrices with the same number of columns: we define $[\delta_{i,j}]_{(m_1+m_2)\times t} = [\alpha_{i,j}]_{m_1\times t} \cup [\beta_{i,j}]_{m_2\times t}$ by $\delta_{i,j} = \alpha_{i,j}$ if $i \leq m_1$ and $\delta_{i,j} = \beta_{i-m_1,j}$ otherwise. It therefore contains all the rows of $[\alpha_{i,j}]_{m_1\times t}$ on the top followed by all the rows of $[\beta_{i,j}]_{m_2\times t}$. For instance,

$$\begin{bmatrix} \alpha_{1,1} & \dots & \alpha_{1,t} \\ \vdots & \ddots & \vdots \\ \alpha_{m_1,1} & \dots & \alpha_{m_1,t} \end{bmatrix} \cup \begin{bmatrix} \beta_{1,1} & \dots & \beta_{1,t} \\ \vdots & \ddots & \vdots \\ \beta_{m_2,1} & \dots & \beta_{m_2,t} \end{bmatrix} = \begin{bmatrix} \alpha_{1,1} & \dots & \alpha_{1,t} \\ \vdots & \ddots & \vdots \\ \alpha_{m_1,1} & \dots & \alpha_{m_1,t} \\ \beta_{1,1} & \dots & \beta_{1,t} \\ \vdots & \ddots & \vdots \\ \beta_{m_2,1} & \dots & \beta_{m_2,t} \end{bmatrix} \tag{7}$$

[1] In the mathematical sense, i.e., the order does not matter

All definitions of E&S were based on mixtures using probability 0.5, such that the decision maker would receive either a risk or a loss for instance. In the definitions of ambiguity apportionment, we will use unions of matrices, such that the decision maker will receive either an ambiguity (represented in a first matrix) or a probability loss (represented in another matrix).

E&S also used addition of two independent lotteries in their definitions. Our second operator, also defined for matrices with the same number of columns, will propose a way to "add" two matrices of mixtures in order to mimic, at the ambiguity level, the addition of independent lotteries. It is a Cartesian sum applied to each column and is defined as followed: $[\alpha_{i,j}]_{m_1\times t} \oplus [\beta_{i,j}]_{m_2\times t} = \bigcup_{k=1}^{m_2} [\alpha_{i,j}+\beta_{k,j}]_{1\leq i\leq m_1, 1\leq j\leq t}$. This $\oplus$-operator adds, one at a time, each row of the right-hand-side matrix to all the rows of the left-hand side matrix. For instance,

$$\begin{bmatrix} \alpha_{1,1} & \dots & \alpha_{1,t} \\ \vdots & \ddots & \vdots \\ \alpha_{m_1,1} & \dots & \alpha_{m_1,t} \end{bmatrix} \oplus \begin{bmatrix} \beta_{1,1} & \dots & \beta_{1,t} \\ \vdots & \ddots & \vdots \\ \beta_{m_2,1} & \dots & \beta_{m_2,t} \end{bmatrix} = \begin{bmatrix} \alpha_{1,1}+\beta_{1,1} & \dots & \alpha_{1,t}+\beta_{1,t} \\ \vdots & \ddots & \vdots \\ \alpha_{m_1,1}+\beta_{1,1} & \dots & \alpha_{m_1,t}+\beta_{1,t} \\ \vdots & \ddots & \vdots \\ \alpha_{1,1}+\beta_{m_2,1} & \dots & \alpha_{1,t}+\beta_{m_2,t} \\ \vdots & \ddots & \vdots \\ \alpha_{m_1,1}+\beta_{m_2,1} & \dots & \alpha_{m_1,t}+\beta_{m_2,t} \end{bmatrix} \tag{8}$$

Note that both $\cup$ and $\oplus$ are not commutative. Brackets will be used to indicate priorities between operators.

3 Ambiguity apportionment

The definitions of the main text will be generalised using matrices. Let x and y be any lotteries such that $x \succsim y$, p is a probability, and $-\kappa < 0$, and define $A_1^{amb} = [-\kappa]_{1\times t}$ and $B_1^{amb} = [0]_{1\times t}$. Preferring $x_{\left(B_1^{amb}\oplus[p]_{1\times t}\right)}y$ to $x_{\left(A_1^{amb}\oplus[p]_{1\times t}\right)}y$ for all $[E_{i,j}]_{1\times t}$ and all initial wealth is a form of stochastic dominance. Recall that in this notation, it is implicitly assumed that $[p-\kappa, p] \subset [0,1]$.

Definition 2. *Define $A_2^{amb} = \left[\varepsilon_{i,j}^1\right]_{2\times t}$ and $B_2^{amb} = [0]_{2\times t}$, with $\varepsilon_{1,j}^1 = -\varepsilon_{2,j}^1$ for all j. A decision maker is ambiguity averse if for all initial wealth levels, for all probabilities p, all such A_2^{amb} and B_2^{amb}, all $x \succsim y$, and all matrices of events $[E_{i,j}]_{2\times t}$, there is at least one symmetric permutation of $[E_{i,j}]_{2\times t}$ (among the two possible ones) such that $x_{\left(B_2^{amb}\oplus[p]_{1\times t}\right)}y$*

is preferred to $x_{\left(A_2^{amb}\oplus[p]_{1\times t}\right)}y$.

This definition of ambiguity aversion captures Ellsberg's original two-urn experiment as a direct test of ambiguity aversion. Recall that the decision maker can choose between these two acts:

$$\text{Option A} = \left\langle \begin{bmatrix} E_0 & \cdots & E_{49} & E_{50} \\ E_{100} & \cdots & E_{51} & \varnothing \end{bmatrix}, 10 \begin{bmatrix} 0 & \cdots & 0.49 & 0.5 \\ 1 & \cdots & 0.51 & 0.5 \end{bmatrix} 0 \right\rangle \tag{9}$$

$$\text{vs. Option B} = \left\langle \begin{bmatrix} E_0 & \cdots & E_{49} & E_{50} \\ E_{100} & \cdots & E_{51} & \varnothing \end{bmatrix}, 10 \begin{bmatrix} 0.5 & \cdots & 0.5 & 0.5 \\ 0.5 & \cdots & 0.5 & 0.5 \end{bmatrix} 0 \right\rangle. \tag{10}$$

The decision maker can then choose between these other two acts:

$$\text{Option A'} = \left\langle \begin{bmatrix} E_{100} & \cdots & E_{51} & \varnothing \\ E_0 & \cdots & E_{49} & E_{50} \end{bmatrix}, 10 \begin{bmatrix} 0 & \cdots & 0.49 & 0.5 \\ 1 & \cdots & 0.51 & 0.5 \end{bmatrix} 0 \right\rangle \tag{11}$$

$$\text{vs. Option B'} = \left\langle \begin{bmatrix} E_{100} & \cdots & E_{51} & \varnothing \\ E_0 & \cdots & E_{49} & E_{50} \end{bmatrix}, 10 \begin{bmatrix} 0.5 & \cdots & 0.5 & 0.5 \\ 0.5 & \cdots & 0.5 & 0.5 \end{bmatrix} 0 \right\rangle. \tag{12}$$

An ambiguity-averse decision maker might choose A and B' (or A' and B) if she has reason to believe that there are more red balls than black balls (or more black balls than red balls), but she would never choose A and A'. This is exactly what the definition says. In the Ellsberg example, the lotteries x and y are degenerate, giving \$10 and \$0 with certainty, respectively.

Definition 3. *Define*

$$A_3^{amb} = ([0]_{2\times t} \oplus [0]_{1\times t}) \cup ([\varepsilon_{i,j}^1]_{2\times t} \oplus [-\kappa]_{1\times t}) = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ \varepsilon_{1,1}^1 - \kappa & \cdots & \varepsilon_{1,t}^1 - \kappa \\ \varepsilon_{2,1}^1 - \kappa & \cdots & \varepsilon_{2,t}^1 - \kappa \end{bmatrix}$$

and $B_3^{amb} = ([0]_{2\times t} \oplus [-\kappa]_{1\times t}) \cup ([\varepsilon_{i,j}^1]_{2\times t} \oplus [0]_{1\times t}) = \begin{bmatrix} -\kappa & \cdots & -\kappa \\ -\kappa & \cdots & -\kappa \\ \varepsilon_{1,1}^1 & \cdots & \varepsilon_{1,t}^1 \\ \varepsilon_{2,1}^1 & \cdots & \varepsilon_{2,t}^1 \end{bmatrix}$, *with* $\varepsilon_{1,j}^1 = -\varepsilon_{2,j}^1$ *for all* j. *A decision maker is ambiguity prudent if for all initial wealth levels, for all probabilities* p, *all such* A_3^{amb} *and* B_3^{amb}, *all* $x \succsim y$, *and all matrices of events* $[E_{i,j}]_{4\times t}$, *there is at least one symmetric permutation of* $[E_{i,j}]_{4\times t}$ *(among the four possible ones)* *such that* $x_{\left(B_3^{amb}\oplus[p]_{1\times t}\right)}y$ *is preferred to* $x_{\left(A_3^{amb}\oplus[p]_{1\times t}\right)}y$.

The definition of ambiguity prudence in the main text only considers $t = 1$ and, therefore, only four mixtures of x and y. The above definition considers combining any series of binary spreads in the probability of obtaining the desired lottery x with fewer chances of obtaining x. The following definition of higher order ambiguity preferences shows how each order can be obtained in a systematic way from lower orders. It is followed by the definition of ambiguity neutrality of order n, which corresponds to a decision maker always preferring an option of type A for some permutations and preferring an option of type B for the others. Strict ambiguity apportionment is then defined.

Definition 4. *Define* $A_n^{amb} = ([0]_{2\times t} \oplus B_{n-2}^{amb}) \cup (\left[\varepsilon_{i,j}^{n\backslash 2}\right]_{2\times t} \oplus A_{n-2}^{amb})$ *and* $B_n^{amb} = ([0]_{2\times t} \oplus A_{n-2}^{amb}) \cup (\left[\varepsilon_{i,j}^{n\backslash 2}\right]_{2\times t} \oplus B_{n-2}^{amb})$, *with* $\varepsilon_{1,j}^{n\backslash 2} = -\varepsilon_{2,j}^{n\backslash 2}$ *for all* j *and* n. *The decision maker exhibits ambiguity apportionment of order* n *if for all initial wealth levels, for all probabilities* p, *all such* A_n^{amb} *and* B_n^{amb}, *all* $x \succsim y$, *and all matrices of events* $[E_{i,j}]_{2^{n-1}\times t}$, *there is at least one symmetric permutation of* $[E_{i,j}]_{2^{n-1}\times t}$ *(among the* 2^{n-1} *possible ones) such that* $x_{\left(B_n^{amb}\oplus[p]_{1\times t}\right)}y$ *is preferred to* $x_{\left(A_n^{amb}\oplus[p]_{1\times t}\right)}y$.

Definition 5. *The decision maker exhibits ambiguity neutrality of order* n *if for all initial wealth levels, for all probabilities* p, *all* A_n^{amb} *and* B_n^{amb} *from definition 4, all* $x \succsim y$, *and all matrices of events* $[E_{i,j}]_{2^{n-1}\times t}$, *there is at least one symmetric permutation of* $[E_{i,j}]_{2^{n-1}\times t}$ *(among the* 2^{n-1} *possible ones) such that* $x_{\left(B_n^{amb}\oplus[p]_{1\times t}\right)}y$ *is preferred to* $x_{\left(A_n^{amb}\oplus[p]_{1\times t}\right)}y$ *and at least one permutation such that* $x_{\left(A_n^{amb}\oplus[p]_{1\times t}\right)}y$ *is preferred to* $x_{\left(B_n^{amb}\oplus[p]_{1\times t}\right)}y$.

Definition 6. *The decision maker exhibits strict ambiguity apportionment of order* n *if she exhibits ambiguity apportionment of order* n *but not ambiguity neutrality of order* n.

4 Ambiguity apportionment and ambiguity models

4.1 Second-order expected utility

Before establishing the results, we provide a definition of n-richness for all n. This condition for $n = 2$ and $n = 4$ is weaker than Richness Assumption 1 in the main text. Therefore, all results provided in this online-appendix that assume n-richness as defined here also hold for the more restrictive richness condition of the main text.

Definition 7. *Let n be any strictly positive natural number. The state space S is said to be n-rich if there exist* $[E_i]_{(n+1)\times t}$*, an initial wealth, and* $x \succ y$ *such that* $\langle \begin{bmatrix} E_1 \\ E_1^c \end{bmatrix}, x \begin{bmatrix} 1 \\ 0 \end{bmatrix} y \rangle \sim \langle \begin{bmatrix} E_i \\ E_i^c \end{bmatrix}, x \begin{bmatrix} 1 \\ 0 \end{bmatrix} y \rangle$ *for all* $i \in \{2, 3, ..., n\}$*, with* E_i^c *the complementary event of* E_i *and either* $E_{(n+1)} = \varnothing$ *or* $E_{(n+1)}$ *can be partitioned into n subsets.*

Proposition 1. *Let n be any strictly positive natural number. Assume second-order expected utility, with u φ strictly increasing and n-times differentiable. Consider the following statements:*

(i) Ambiguity apportionment of order n.

(ii) $sgn\left(\varphi^{(n)}\right) = (-1)^{n+1}$.

Statement (i) is necessary for statement (ii). It is also sufficient if S is 2^{n-1}*-rich.*

Proof. Assume that $sgn\left(\varphi^{(n)}\right) = (-1)^{n+1}$, and consider any $x_{\left(A_n^{amb}\oplus[p]_{1\times t}\right)}y$ and $x_{\left(B_n^{amb}\oplus[p]_{1\times t}\right)}y$. Let us scale u (which is defined up to scale and unit) such that $Eu(\omega + y) = 0$ and $Eu(\omega + x) = 1$. Let $\alpha_{i,j}$ and $\beta_{i,j}$ refer to the elements of A_n^{amb} and B_n^{amb}. Then $Eu(\omega + x_{(p+\alpha_{i,j})}y) = p + \alpha_{i,j}$ and $Eu(\omega + x_{(p+\beta_{i,j})}y) = p + \beta_{i,j}$. Moreover, if we assign $\frac{1}{m}$ to all $\alpha_{i,j}$ for a given j, then we obtain an A_n^{risk}. The same can be done for the $\beta_{i,j}$s. From the theorem of E&S (applied to φ with p playing the role of the initial wealth), it follows that for all $j \in \{1, \cdots, t\}$, $\sum_{i=1}^m \frac{1}{m}\varphi(p + \alpha_{i,j}) \leq \sum_{i=1}^m \frac{1}{m}\varphi(p + \beta_{i,j})$. It implies
$\left(\sum_{i=1}^m P(E_{i,j})\right)\left(\sum_{i=1}^m \varphi(p + \alpha_{i,j})\right) \leq \left(\sum_{i=1}^m P(E_{i,j})\right)\left(\sum_{i=1}^m \varphi(p + \beta_{i,j})\right)$
for all $j \in \{1, \cdots, t\}$. Hence,
$\sum_{j=1}^t \left(\left(\sum_{i=1}^m P(E_{i,j})\right)\left(\sum_{i=1}^m \varphi(p + \alpha_{i,j})\right)\right) \leq \sum_{j=1}^t \left(\left(\sum_{i=1}^m P(E_{i,j})\right)\left(\sum_{i=1}^m \varphi(p + \beta_{i,j})\right)\right)$. It is then enough to see that the left-hand side of the inequality is the sum of the second-order expected utility of all symmetric permutations of act $\langle [E_{i,j}]_{m\times t}, x_{\left(A_n^{amb}\oplus[p]_{1\times t}\right)}y\rangle$ and that the right-hand side is the sum of the second-order expected utility of all symmetric permutations of act $\langle [E_{i,j}]_{m\times t}, x_{\left(B_n^{amb}\oplus[p]_{1\times t}\right)}y\rangle$ to conclude that all symmetric permutations of $\langle [E_{i,j}]_{m\times t}, x_{\left(A_n^{amb}\oplus[p]_{1\times t}\right)}y\rangle$ cannot be strictly preferred to their respective permutation of $\langle [E_{i,j}]_{m\times t}, x_{\left(B_n^{amb}\oplus[p]_{1\times t}\right)}y\rangle$. Ambiguity apportionment of order n holds.

Assume that $sgn\left(\varphi^{(n)}\right) = (-1)^n$ (and $\varphi^{(n)} \neq 0$) on $[\eta - \alpha - \kappa, \eta + \alpha]$ (for some nonzero η, α, and κ). From the continuity of u, there exists an outcome β such that $u(\beta) = \eta - \alpha$

and another outcome γ such that $u(\gamma) = \eta + \alpha$. Let x and y be the degenerate lotteries that give β and γ, respectively. Then, the utility of $x_{\left[\frac{1}{2}\right]}y$ is η. Define $\left[\varepsilon_i^j\right]_2$ for all $j \in \{1, ..., n \setminus 2\}$ as follows: $\varepsilon_1^j = -\varepsilon_2^j = \frac{\alpha}{n\setminus 2}$. Together with κ, the $\left[\varepsilon_i^j\right]_2$s allow us to define a B_n^{amb} and its corresponding A_n^{amb}, and then $x_{\left(B_n^{amb} \oplus \left[\frac{1}{2}\right]\right)}y$ and $x_{\left(A_n^{amb} \oplus \left[\frac{1}{2}\right]\right)}y$. The lottery that gives each element of B_n^{amb} (A_n^{amb}) the same probability can be written as a B_n^{risk} (A_n^{risk}), with each $\left[\varepsilon_i^j\right]_2$ becoming a zero-mean lottery and the $\cup$s becoming mixture with probability $\frac{1}{2}$. From E&S, it can be derived that the sum of the φ values of the elements of A_n^{risk} (and thus of A_n^{amb}) is strictly higher than those of the elements of B_n^{risk} (and thus of B_n^{amb}). There are 2^{n-1} elements in A_n^{amb}. Because S is 2^{n-1}-rich and assuming that second-order expected utility holds, we can find a partition $[E_i]_{(2^{n-1}+1)}$ of S in 2^{n-1} equally likely events plus another (possibly empty) event $E_{(2^{n-1}+1)}$. If $E_{(2^{n-1}+1)} = \varnothing$, we can thus remove it from the partition and conclude that for all symmetric permutation $\boldsymbol{\sigma}$, $\langle \boldsymbol{\sigma}\left([E_i]_{2^{n-1}}\right), x_{\left(A_n^{amb} \oplus \left[\frac{1}{2}\right]\right)}y \rangle$ is strictly preferred to $\langle \boldsymbol{\sigma}\left([E_i]_{2^{n-1}}\right), x_{\left(B_n^{amb} \oplus \left[\frac{1}{2}\right]\right)}y \rangle$. Therefore, ambiguity apportionment of order n does not hold. If $E_{(2^{n-1}+1)} \neq \varnothing$, let us build the matrix $[F_{i,j}]_{2^{n-1}\times 2}$ such that $F_{i,1} = E_i$ and $F_{i,2} \subset E_{(2^{n-1}+1)}$ for all $i \in \{1, \cdots, 2^{n-1}\}$. Define $A_n^{amb(2)}$ and $B_n^{amb(2)}$ as two-column matrices containing A_n^{amb} and B_n^{amb}, respectively, in the first column and 0 in the second column. We can thus conclude that for all symmetric permutations $\boldsymbol{\sigma}$, $\langle \boldsymbol{\sigma}\left([F_{i,j}]_{2^{n-1}\times 2}\right), x_{\left(A_n^{amb(2)} \oplus \left[\frac{1}{2}\right]_{1\times 2}\right)}y \rangle$ is strictly preferred to $\langle \boldsymbol{\sigma}\left([F_{i,j}]_{2^{n-1}\times 2}\right), x_{\left(B_n^{amb(2)} \oplus \left[\frac{1}{2}\right]_{1\times 2}\right)}y \rangle$. Therefore, ambiguity apportionment of order n does not hold.

□

4.2 Multiplier preferences

Proposition 2. *If preferences are multiplier preferences, then they exhibit ambiguity apportionment of order n for all n. If S is 2^{n-1}-rich and $\theta < \infty$, then strict apportionment of order n holds.*

Proof. Multiplier preferences are ordinally equivalent to second-order expected utility with φ_θ as defined in Eq.1. From the derivatives of the exponential function, we conclude that $sgn(\varphi_\theta^{(n)}) = (-1)^{n+1}$ for all n. From Theorems A.1, we know that it implies ambiguity apportionment of order n for all n. (Note that the richness condition of Theorem A.1 is only used for the opposite implication).

If $\theta < \infty$, the successive derivatives of φ_θ are not zero. Consider any B_n^{amb} and its corresponding A_n^{amb}, which are developed such that the elements of $\left[\varepsilon_{i,j}^{n\backslash 2}\right]_{2\times t}$ (from definition 10) are never 0 and with $t = 1$. Using the 2^{n-1}-richness of the state space, we can find a partition $[E_i]_{(2^{n-1}+1)}$ of S in 2^{n-1} equally likely events (in terms of μ) plus another (possibly empty) event $E_{(2^{n-1}+1)}$. If $E_{(2^{n-1}+1)} = \varnothing$, remove it from the partition. Such acts mimic for φ_θ what E&S's B_n^{risk} and A_n^{risk} did for u. We can therefore conclude that for any symmetric permutation of this new partition, the act based on B_n^{amb} will be strictly preferred to the act based on A_n^{amb}. Therefore ambiguity neutrality of order n does not hold, and strict ambiguity apportionment of order n does. If $E_{(2^{n-1}+1)} \neq \varnothing$, let us build the matrix $[F_{i,j}]_{2^{n-1}\times 2}$ such that $F_{i,1} = E_i$ and $F_{i,2} \subset E_{(2^{n-1}+1)}$ for all $i \in \{1, \cdots, 2^{n-1}\}$. Define $A_n^{amb(2)}$ and $B_n^{amb(2)}$ as two-column matrices containing A_n^{amb} and B_n^{amb} in the first column and 0 in the second column. We can thus conclude that for all symmetric permutations, the act based on $B_n^{amb(2)}$ will be strictly preferred to $A_n^{amb(2)}$. Therefore, ambiguity neutrality of order n does not hold, and strict ambiguity apportionment of order n does. □

5 Ambiguity models with non-expected utility under risk

All of the ambiguity models considered in the main text and in this online appendix assumed expected utility under risk. This assumption may be violated empirically, for instance by the paradox proposed by Allais (1953). Abdellaoui and Zank (2014), for instance, consider the extensions of second-order expected utility and Choquet expected utility to preferences satisfying rank-dependent utility under risk (Quiggin, 1982). One special case of rank-dependent utility is the neo-additive version (Cohen, 1992; Webb and Zank, 2011). A lottery x would be evaluated by $(1-a')Eu(\omega + x) + \frac{a'-b'}{2}max\{u(\alpha) : q(x=\alpha) > 0\} + \frac{a'+b'}{2}min\{u(\alpha) : q(x=\alpha) > 0\}$ (where q denotes the objective probabilities assigned by x to the various outcomes). In the proofs of all results, we use the equality $Eu(x_p y) = pEu(x) + (1-p)Eu(y)$. The same equality holds for neo-additive rank dependent utility if x and y have the same best and worst outcomes. It is therefore sufficient to adapt the definition of ambiguity prudence by requiring that x and y have the same best and worst outcomes to derive the same results as in the paper and in this online appendix, but with neo-additive rank-dependent utility under risk.

6 Precautionary savings

6.1 Precautionary savings with the smooth model

To understand the implications of the various types of prudence, let us consider a stylised model derived from Kimball's (1990) model and add ambiguity to it. Consider a consumer who can decide how much of her total wealth she will consume in each of two periods. For simplicity, let us ignore discounting and assume that she has the same utility for both periods so that other sources of saving, such as life-cycle saving, do not interact with precautionary motives. The consumer therefore chooses her saving level α in period 1 to maximise

$$u(\omega - \alpha) + u(\alpha). \tag{13}$$

Assuming risk aversion (and therefore $u'' < 0$) and monotonicity, the solution to this problem is obviously $\alpha = \frac{\omega}{2}$. Kimball (1990) studied precautionary saving by adding the possibility of a zero-mean risk x_0 occurring at period 2, so that the consumer would now maximise

$$u(\omega - \alpha) + Eu(\alpha + x_0). \tag{14}$$

Because the first-order condition becomes $u'(\omega - \alpha) = Eu'(\alpha + x_0)$, he concluded that a risk-averse consumer will increase her precautionary saving if she is prudent (because $u''' > 0$ is equivalent to $u'(\frac{\omega}{2}) < Eu'(\frac{\omega}{2} + x_0)$ for all ω and x_0). Crainich et al. (2013) showed that the same results hold for risk lovers.

Let us now assume that the consumer's preferences are represented by a two-stage expected utility model and study her saving α when there is ambiguity about her future income. Her income in period 2 might increase or decrease by ε. The probability of the increase is unknown to the consumer and can be $\frac{1}{2} - \eta$ or $\frac{1}{2} + \eta$. For simplicity, we will assume that the consumer has complete ignorance about which probability is the correct one and acts as if they are equally likely. Furthermore, φ and u are assumed to be strictly increasing. The consumer will maximise

$$\varphi\left(u(\omega - \alpha)\right) + \frac{1}{2}\varphi\left(\left(\frac{1}{2} - \eta\right)u(\alpha + \varepsilon) + \left(\frac{1}{2} + \eta\right)u(\alpha - \varepsilon)\right) \\ + \frac{1}{2}\varphi\left(\left(\frac{1}{2} + \eta\right)u(\alpha + \varepsilon) + \left(\frac{1}{2} - \eta\right)u(\alpha - \varepsilon)\right). \tag{15}$$

The first term corresponds to period 1. It is certain, but the consumer applies both the risky utility function u and the ambiguity function φ. The rest corresponds to the second period. If the real probability of increase is $\left(\frac{1}{2}-\eta\right)$, which can occur with probability $\frac{1}{2}$ according to the consumer, she will obtain an expected utility of $\left(\frac{1}{2}-\eta\right)u(\alpha+\varepsilon)+\left(\frac{1}{2}+\eta\right)u(\alpha-\varepsilon)$. Because the real probability is unknown, she transforms this expected utility with her ambiguity function φ and computes the expectation.

As a benchmark, we begin with no future risk ($\varepsilon=0$) and, hence, no ambiguity. Equation 15 reduces to

$$\varphi\left(u(\omega-\alpha)\right)+\varphi\left(u(\alpha)\right), \tag{16}$$

A sufficient condition for the existence of saving ($\alpha \geq 0$) is risk aversion and ambiguity aversion ($u'', \varphi'' < 0$). From now on, we therefore assume risk aversion and ambiguity aversion and conclude that the optimal saving level is $\frac{\omega}{2}$. We can now introduce a future risk ($\varepsilon > 0$) but still no ambiguity ($\eta = 0$). Equation 15 becomes

$$\varphi\left(u(\omega-\alpha)\right)+\varphi\left(\frac{1}{2}u(\alpha+\varepsilon)+\frac{1}{2}u(\alpha-\varepsilon)\right). \tag{17}$$

The optimal saving level α^* is given by the first-order condition

$$\varphi'\left(u(\omega-\alpha)\right)u'(\omega-\alpha)=\varphi'\left(\frac{1}{2}u(\alpha+\varepsilon)+\frac{1}{2}u(\alpha-\varepsilon)\right)\times\left(\frac{1}{2}u'(\alpha+\varepsilon)+\frac{1}{2}u'(\alpha-\varepsilon)\right). \tag{18}$$

Risk and ambiguity aversion imply that $\varphi'\left(u(\frac{\omega}{2})\right) \leq \varphi'\left(\frac{1}{2}u(\frac{\omega}{2}+\varepsilon)+\frac{1}{2}u(\frac{\omega}{2}-\varepsilon)\right)$. If the agent is risk prudent (i.e., u' is convex), then $u'(\frac{\omega}{2}) \leq \frac{1}{2}u'(\frac{\omega}{2}+\varepsilon)+\frac{1}{2}u'(\frac{\omega}{2}-\varepsilon)$. Moreover, all of these terms are positive (u and φ are strictly increasing). Therefore, for $\alpha=\frac{\omega}{2}$, the left-hand member of equation 18 is less than or equal to the right-hand one. Therefore, because φ' and u' are decreasing, $\alpha^* \geq \frac{\omega}{2}$. Risk prudence is sufficient for future risks to trigger an increase of precautionary saving.[2]

We can now add some ambiguity ($\varepsilon > 0$ and $\eta > 0$) and study the new optimal saving level α^{**}. The first-order condition of equation 15 is

$$\begin{aligned}&\varphi'\left(u(\omega-\alpha)\right)u'(\omega-\alpha)=\\&\quad\frac{1}{2}\varphi'\left(\left(\frac{1}{2}-\eta\right)u(\alpha+\varepsilon)+\left(\frac{1}{2}+\eta\right)u(\alpha-\varepsilon)\right)\times\left(\left(\frac{1}{2}-\eta\right)u'(\alpha+\varepsilon)+\left(\frac{1}{2}+\eta\right)u'(\alpha-\varepsilon)\right)\\&+\frac{1}{2}\varphi'\left(\left(\frac{1}{2}+\eta\right)u(\alpha+\varepsilon)+\left(\frac{1}{2}-\eta\right)u(\alpha-\varepsilon)\right)\times\left(\left(\frac{1}{2}+\eta\right)u'(\alpha+\varepsilon)+\left(\frac{1}{2}-\eta\right)u'(\alpha-\varepsilon)\right).\end{aligned} \tag{19}$$

We can show that an ambiguity-prudent consumer will have to increase her saving with respect to α^*. If we apply the spread in probabilities $\pm\eta$ to $\varphi'\left(\frac{1}{2}u(\alpha^*+\varepsilon)+\frac{1}{2}u(\alpha^*-\varepsilon)\right)$

[2]It is however not a necessary condition. See Osaki and Schlesinger (2014) for a discussion.

in equation 18 (which determined α^*), ambiguity prudence will favor the right member of the equation (the convexity of φ' 'discounts' the marginal utility of the non-ambiguous period 1 relative to the ambiguous period 2). As a consequence, the saving level will have to be increased to restore the equality. Risk and ambiguity aversion will ensure that the changes from equation 18 to equation 19 can only reinforce this effect. Overall, we obtain $\alpha^{**} \geq \alpha^*$. Ambiguity prudence thus implies that future ambiguity triggers an increase of precautionary saving, exactly as risk prudence implied that a future risk increases precautionary saving.

To formally establish the result, we start from equation 19 and show that α^* (the no-ambiguity saving level) may not satisfy the equality. We show that α^{**} has to be more than α^* to restore the equality.

$$\varphi'\left(u(\omega-\alpha^*)\right)u'(\omega-\alpha^*)$$

$$\leq$$

$$\left(\varphi'\left(\left(\frac{1}{2}-\eta\right)u(\alpha^*+\varepsilon)+\left(\frac{1}{2}+\eta\right)u(\alpha^*-\varepsilon)\right)+\varphi'\left(\left(\frac{1}{2}+\eta\right)u(\alpha^*+\varepsilon)+\left(\frac{1}{2}-\eta\right)u(\alpha^*-\varepsilon)\right)\right) \times \frac{1}{2}\times\left(\frac{1}{2}u'(\alpha^*+\varepsilon)+\frac{1}{2}u'(\alpha^*-\varepsilon)\right) \quad (20)$$

$$\leq$$

$$\frac{1}{2}\varphi'\left(\left(\frac{1}{2}-\eta\right)u(\alpha^*+\varepsilon)+\left(\frac{1}{2}+\eta\right)u(\alpha^*-\varepsilon)\right)\times\left(\left(\frac{1}{2}-\eta\right)u'(\alpha^*+\varepsilon)+\left(\frac{1}{2}+\eta\right)u'(\alpha^*-\varepsilon)\right) + \frac{1}{2}\varphi'\left(\left(\frac{1}{2}+\eta\right)u(\alpha^*+\varepsilon)+\left(\frac{1}{2}-\eta\right)u(\alpha^*-\varepsilon)\right)\times\left(\left(\frac{1}{2}+\eta\right)u'(\alpha^*+\varepsilon)+\left(\frac{1}{2}-\eta\right)u'(\alpha^*-\varepsilon)\right). \quad (21)$$

Inequality 20 is derived from the first-order condition of α^* (equation 18) and from the convexity of φ' (i.e., ambiguity prudence). Inequality 21 is a consequence of risk and ambiguity aversion. Indeed, the right-hand side of inequality 21 can be written as the left-hand side plus $\left(\varphi'\left(\left(\frac{1}{2}-\eta\right)u(\alpha^*+\varepsilon)+\left(\frac{1}{2}+\eta\right)u(\alpha^*-\varepsilon)\right)-\varphi'\left(\left(\frac{1}{2}+\eta\right)u(\alpha^*+\varepsilon)+\left(\frac{1}{2}-\eta\right)u(\alpha^*-\varepsilon)\right)\right)\times\left(\eta u'(\alpha^*-\varepsilon)-\eta u'(\alpha^*+\varepsilon)\right)$. Risk aversion implies that $u'(\alpha^*-\varepsilon) \geq u'(\alpha^*+\varepsilon)$ and therefore, $\eta u'(\alpha^*-\varepsilon)-\eta u'(\alpha^*+\varepsilon) \geq 0$. Ambiguity aversion (and u being increasing) implies that $\varphi'\left(\left(\frac{1}{2}-\eta\right)u(\alpha^*+\varepsilon)+\left(\frac{1}{2}+\eta\right)u(\alpha^*-\varepsilon)\right) \geq \varphi'\left(\left(\frac{1}{2}+\eta\right)u(\alpha^*+\varepsilon)+\left(\frac{1}{2}-\eta\right)u(\alpha^*-\varepsilon)\right)$. Inequality 21 follows. From inequalities 20 and 21 and knowing that φ' and u' are decreasing, we obtain $\alpha^{**} \geq \alpha^*$.

6.2 An alternative model of precautionary savings

Below, a model of precautionary saving under ambiguity based on Choquet expected utility is introduced. In such a case, ambiguity prudence always hold and it does not allow to fully study its impact on precautionary saving. Yet, it is shown that, when strict ambiguity prudence holds, ambiguity aversion is then necessary and sufficient for ambiguity to increase the consumer's precautionary saving. Intuitively, the presence of strict ambiguity prudence is automatic when the consumer simply cares about ambiguity. It implies that the consumer considers the best and the worst expected utility she may get. Ambiguity aversion is then equivalent to assigning more weight to the worst case, which in terms of first order condition means to the highest expected marginal utility. Starting from the optimal saving in the non-ambiguous case, an increase of saving is then required to restore the equality in the first-order condition. The same result holds for α-maxmin with ϵ-contamination. The ϵ-contamination automatically implies ambiguity prudence and means that the consumer perceives ambiguity. Ambiguity aversion will also give more weight to the worst case and be necessary and sufficient for an increase of precautionary saving.

We start from Equation 13 above and introduce a binary risk.

$$u(\omega-\lambda)+\frac{1}{2}u(\lambda+\varepsilon)+\frac{1}{2}u(\lambda-\varepsilon). \tag{22}$$

The optimal saving level λ^* is given by the first-order condition

$$u'(\omega-\lambda)=\frac{1}{2}u'(\lambda+\varepsilon)+\frac{1}{2}u'(\lambda-\varepsilon). \tag{23}$$

As discussed above, if $u'''\geq 0$, $\lambda^*\geq\frac{\omega}{2}$. In the remainder of this subsection, we assume $u''<0$ and $u'''>0$. Let us introduce ambiguity about the future income variation $\pm\varepsilon$. The probability of the increase is unknown to the consumer and can be $\frac{1}{2}-\eta$ or $\frac{1}{2}+\eta$, depending on an event E (and with $\eta>0$). Assume Choquet expected utility with W neo-additive and $P(E)=\frac{1}{2}$. The consumer now maximises:

$$\begin{aligned} &u(\omega-\lambda)+(1-a)\left[\frac{1}{2}u(\lambda+\varepsilon)+\frac{1}{2}u(\lambda-\varepsilon)\right] \\ &\quad+\frac{a-b}{2}\left[\frac{1+2\eta}{2}u(\lambda+\varepsilon)+\frac{1-2\eta}{2}u(\lambda-\varepsilon)\right]+\frac{a+b}{2}\left[\frac{1-2\eta}{2}u(\lambda+\varepsilon)+\frac{1+2\eta}{2}u(\lambda-\varepsilon)\right] \end{aligned} \tag{24}$$

First note that a must be strictly positive (implying strict ambiguity prudence). If it is null, η would play no role anymore (remember b is restricted to $[-a, a]$) and the saving level would remain λ^*. The first order condition becomes:

$$u'(\omega - \lambda) = \frac{1 - 2b\eta}{2} u'(\lambda + \varepsilon) + \frac{1 + 2b\eta}{2} u'(\lambda - \varepsilon). \quad (25)$$

Assume $b \geq 0$ (which is equivalent to strict ambiguity aversion). Since u' is decreasing, $u'(\omega - \lambda^*) \leq \frac{1-2b\eta}{2} u'(\lambda^* + \varepsilon) + \frac{1+2b\eta}{2} u'(\lambda^* - \varepsilon)$. Hence the new optimal saving level λ^{**} must satisfy $\lambda^{**} \geq \lambda^*$. By symmetry of the argument, we conclude $b \leq 0$ implies $\lambda^{**} \leq \lambda^*$. In this model, if $a > 0$, then future ambiguity will increase the consumer's precautionary saving if and only if the agent is ambiguity averse.

This model could be rewritten using α-maxmin with ϵ-contamination. In such a case, $\epsilon > 0$ (implying strict ambiguity prudence) is necessary for ambiguity to have an impact on precautionary savings and $\alpha \geq \frac{1}{2}$ (equivalent to ambiguity aversion) is also equivalent to the future ambiguity increasing the consumer's precautionary saving.

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